

©Fab Fernandez/Image Source/Getty Images RF

Introduction to Probability

Learning Objectives

- Describe fundamental probability concepts.
- Apply the rules of probability.
- Distinguish between independent and dependent events.
- Calculate and interpret probabilities from a contingency table.
- Apply the total probability rule.
- Apply Bayes' theorem.
- Use a counting rule to calculate the probability of an event.

Learning Objectives

You will learn:

- Basic probability concepts
- Conditional probability
- To use Bayes' Theorem to revise probabilities

A **probability** is a numerical value that measures the likelihood that an uncertain event occurs.

The value of a probability is between zero (0) and one (1).

- ✓ A probability of zero indicates *impossible* events.
- ✓ A probability of one indicates *definite* events.

KEY TERMS

Random Experiment

Well-defined process of observing a given chance phenomena through a series of trials each of which leads to a single outcome.

Event

- Outcome of an experiment or a result of a trial.

Example	Experiment	Outcome
1	Writing an exam	Pass or Fail
2	Tossing a coin	Getting Head or Tail
3	Observing a machine	Working or not working
4	Throwing a die	Getting no. 1,2,3,..6.

KEY TERMS

- **Mutually exclusive event**

  - When both events cannot happen simultaneously.
  - Happening of one event prevents the happening of the other event and vice-versa.

  Example – **In tossing a coin, either head or tail will appear . Both cannot happen at the same time.**

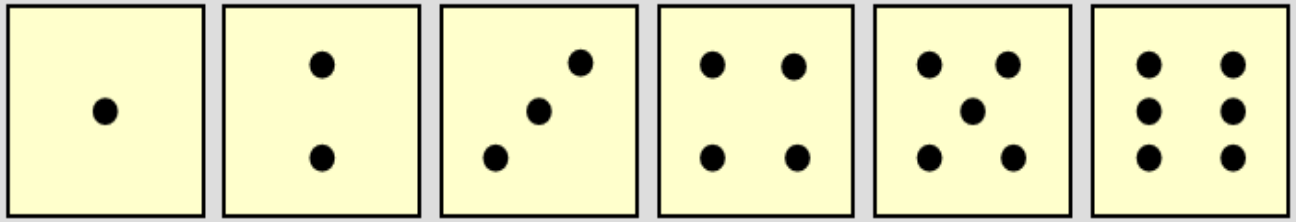
- **Not mutually exclusive event**

  - When both events can occur simultaneously.

  Example – **Driving a car and listening to music.**

The Sample Space is the collection of all possible events

ex. All 6 faces of a die:



ex. All 52 cards in a deck of cards

ex. All possible outcomes when having a child:

Boy or Girl

KEY TERMS

- **Collectively exhaustive event (Sample space)**

  - Inclusion of all possible outcomes.

  Example – **Tossing a coin – Head, Tail**

  - Throwing a die. Getting number 1,2,3,4,5 and 6.

- **Dependent event**

  - Event which affect the probability of occurrence of the other event.

  Example – **Getting admission letter depends on getting interview card.**

- **Independent event**

  - Events which do not affect the probability of occurrence of the other event.

  Example – **Getting an offer letter from company A is independent of getting an offer letter from company B**

  - Getting head in the second toss is independent of getting head in the first toss.

KEY TERMS

- **Complimentary event** : Probability of non-occurrence of an event.
- **Equally likely event** : In a sample space containing atleast 2 events,the probability of occurrence of each of the event is same
- **Marginal Probability** : Probability of occurrence of a single event
- **Joint Probability** : **P**robability of occurrence of two or more events.
- **Contingency table** : Observations are classified on the basis of 2 variables and arranged in a table, the resulting table is referred to as contingency table

There are three approaches to assessing the probability of an uncertain event:

1. *a priori -- based on prior knowledge of the process*

$$\text{probability of occurrence} = \frac{X}{T} = \frac{\text{number of ways in which the event occurs}}{\text{total number of possible outcomes}}$$

Assuming all outcomes are equally likely

2. empirical probability

$$\text{probability of occurrence} = \frac{\text{number of ways in which the event occurs}}{\text{total number of possible outcomes}}$$

3. subjective probability

based on a combination of an individual's past experience, personal opinion, and analysis of a particular situation.

The Classical Approach

The classical approach to assigning probability to a particular event is based on the assumption of equally likely events.

An event is characterized to be equally likely event if there is equal chance of occurring an event.

The probability of an event in classical approach is given as follows:

$$\text{Probability of an event} = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of all possible outcomes}}$$

It is important to note that classical approach to probability is also called *apriori approach to probability* as one can determine probability of an event in advance logically if the events are equally likely events.

This approach to probability is useful in games of chances where the assumption of equally likely event is sound sensible but in business this assumption is not appropriate.

Relative frequency approach

The relative frequency approach to assigning probability of event requires historical data on an event.

Under this approach, probability of an event is based on how frequently a particular event has occurred in the past.

It is defined as the long run relative frequency with which an event occurs.

$$\text{Probability of an event} = \frac{Number\ \ of\ \ times\ \ occured\ \ \ in\ the\ \ past}{Total\ \ number\ \ \ of\ \ observtion}$$

Suppose the number of withdrawals on an average was 25 at a particular bank over the last 200 days. The relative frequency of 25 withdrawals on any day is 25/200 or 12.5 percent. In other words, there is 12.5% chance of 25 withdrawals at this bank on any day in the long run.

Subjective Probability

This is developed to deal with limitations of the earlier approaches.

One, if the events are not equally likely then one cannot determine the probability with classical approach.

Second, if the past data is not available on an event then one cannot determine probability based on relative frequency approach.

Under such circumstances, one can resort to subjective probability. It refers to the degree of belief a particular person attach to some event.

Subjective probability differs from person to person.

For example, what is the probability that Russia, China and India will form a strategic group? What is the probability that India will become a permanent member of Security Council in UN in coming 5 years?

How to assign probability to such event? In many cases, opinion of the experts or personal expertise of particular field may help to estimate probabilities values.

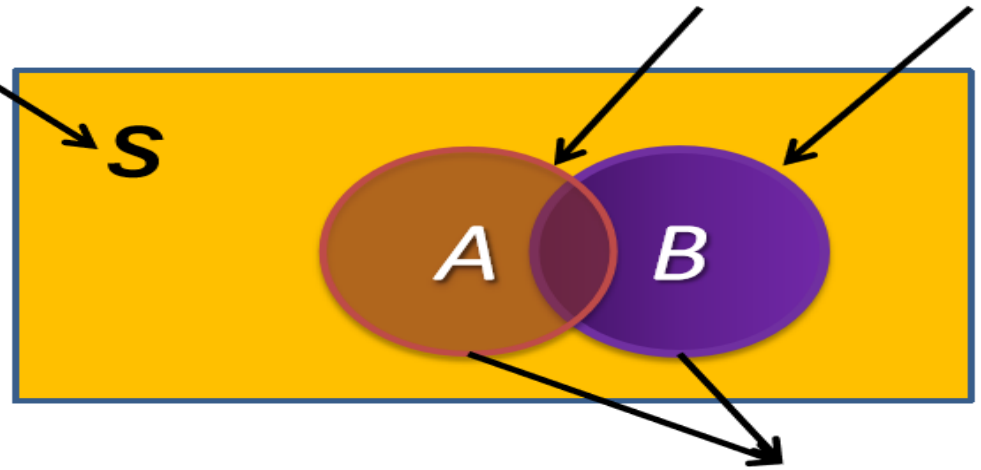
NOTATIONS

• Probability of occurrence of event A.	P (A)	Read as P of (A)
• Probability of non-occurrence of event A. Always P (A) + P ($\bar{A}$) = 1	P ($\bar{A}$)or P(A')	Read as P of A complement
• Probability of occurrence of both the events (A and B).	P (AB)	Read as P of AB
• Probability of non-occurrence of an event A and B.	**P (A' B')**	**Read as P of A complement & B complement**
• Probability of occurrence of an event A or B.	P(A + B)	Read as P of A plus B
• Probability of occurrence of an event A given that B has occured	P(A I B)	Read as P of A given B

In the conditional probability statement, the symbol "I" means "given."
Whatever follows "I" has already occurred.

A *Venn Diagram* represents the *sample space* for the event(s).

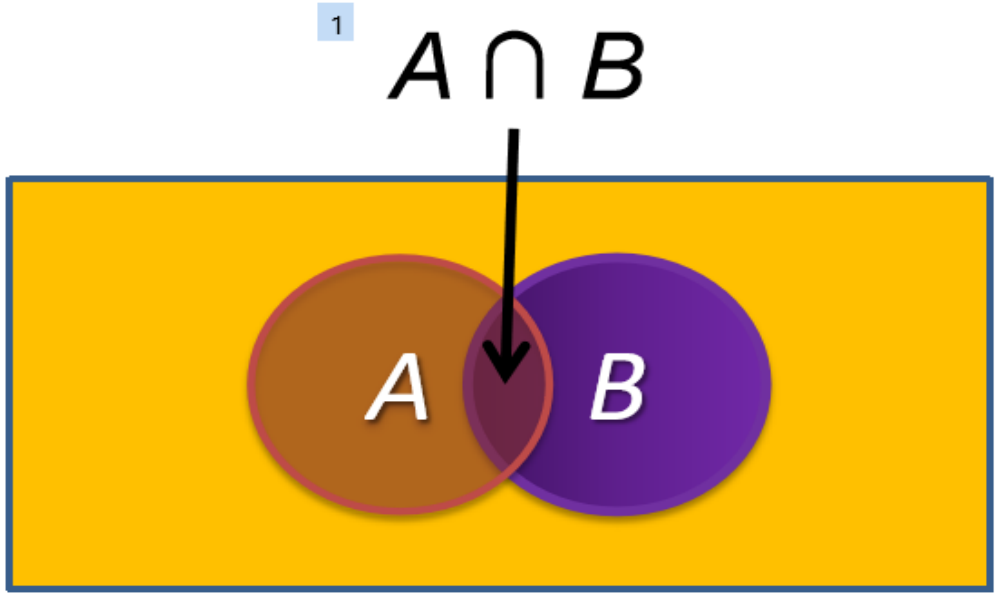
- ✓ For example, this Venn Diagram illustrates the sample space for events *A* and *B*.



- ✓ The **union** of two events ($A \cup B$) is the event consisting of all simple events in *A* or *B*.

A **sample space** includes all possible outcomes of the experiment.

The **intersection** of two events ($A \cap B$) consists of all outcomes in A and B.



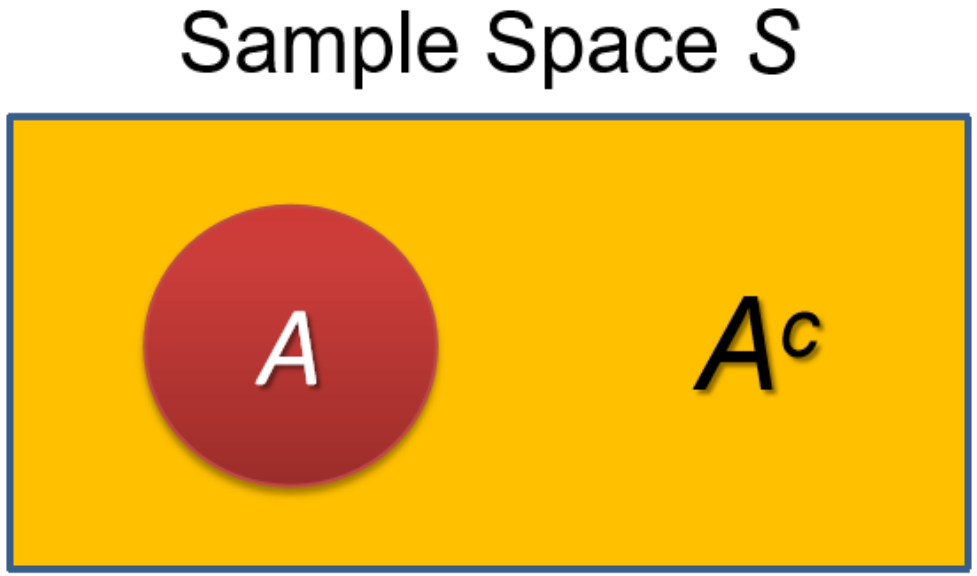
- The **complement** of event A (i.e., A^c) is the event consisting of all outcomes in the sample space S that are not in A.



The **Complement Rule**

- ✓ The probability of the complement of an event, $P(A^c)$, is equal to one minus the probability of the event.

$$P\left(A^c\right) = 1 - P\left(A\right)$$



ADDITION RULE

- ***There are situations where you may wish to evaluate the probability that 2 or more of several events will occur in an experiment.***

Addition rule aids in the evaluation of such probabilities.

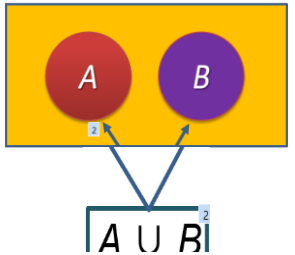
- **Addition Rule for mutually exclusive**

- If A & B are mutually exclusive, then

$$P(A + B) = P(A) + P(B)$$

  - If there are three events, say A, B, C, which are mutually exclusive, then

$$P(A + B + C) = P(A) + P(B) + P(C)$$



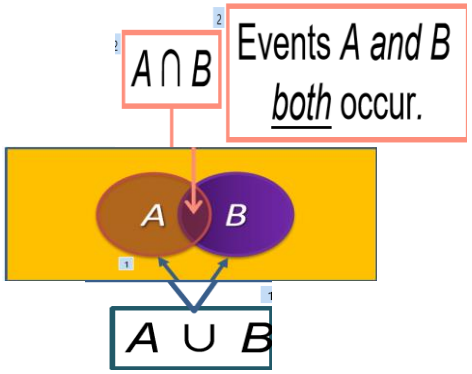
- **Addition Rule for not mutually exclusive**

- If A and B are not mutually exclusive, then

$$P(A + B) = P(A) + P(B) - P(AB)$$

- If there are three events, A, B, C, which are not mutually exclusive then

$$P(A + B + C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$



Addition Rule (Mutually Exclusive Events)

- If two events A and B are mutually exclusive, the probability of either A or B happening is the sum of their respective probabilities.

$$P(A \cup B) = P(A) + P(B)$$

Read as A union B same as plus

- Suppose that I throw a die and I want to find the probability of getting either 3 dots (event A) or 4 dots (event B).

These events are mutually exclusive because if the die throws up 3, the occurrence of 4 is ruled out.

Therefore, the probability of getting either event A or event B is the sum of individual probabilities

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{1}{6}$$

$$P(A \cup B) = P(A) + P(B)$$

P(AUB) Read as A union B same as P(A+B)

$$P(AUB) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = 0.33$$

Thus the probability of getting either 3 or 4 is 0.33 or 33%.

Addition Rule (Non-mutually exclusive events)

- If two events are non-mutually exclusive then the occurrence of either A or B or both A and B. The probability of either A or B is:

P(A ∩ B) **Read as A intersection B same as P(AB)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Problem 1

The probability of a contractor getting a plumbing contract is 2/3 and the probability that he will not get an electrical contract is 4/9. If the probability of getting atleast one contract is 4/5, what is the probability that he will get both

Probability of getting a plumbing contract	P(A)	2/3
Probability of not getting an electrical contract	P(B ')	4/9 P(B)= 1-4/9=5/9
Probability of getting atleast one contract	P(A+B)	4/5
Probability of getting both the contracts	P(AB)	?

Since the events are not mutually exclusive, P(A+B) = P(A) + P(B) – P(AB)

P(AB)= P(A) + P(B) – P(A + B) = 2/3 +5/9 -4/5 =19 /45

PROBLEM - 2

- **A construction company is bidding for two contracts, A and B. The probability that the company will get contract A is 3/5, the probability that the company will get contract B is 1/3 and the probability that the company will get both contracts is 1/8. What is the probability that the company will get contract A or B?**

Data Given

Let P (A) be the probability of getting contract A be 3/5

Let P (B) be the probability of getting contract B be 1/3

Let P (AB) be the probability of getting both the contracts be 1/8

Find

the Probability that the company will get contract A or B: P(A+B)

Since the events are not mutually exclusive then

$$P(A + B) = P(A) + P(B) - P(AB)$$

$$P(A + B) = 3/5 + 1/3 - 1/8 = 0.6 + 0.33 - 0.125 = 0.805$$

ie. Probability of getting contract A or contract B is 0.805. (i.e) 80.5% chance of getting a contract A or contract B

PROBLEM - 3

- **If the probability is 0.30 that a management accountant's job applicant has a post graduate degree, 0.70 that he has some work experience as a Chief Financial Accountant, and 0.20 that he both. Out of 200 applicants, approximately, what number would have either a post graduate degree or some professional work experience?**

Data Given

Let P(A) be the probability of management accountant having a PG degree	0.30
Let P(B) be the probability of management accountant having work experience	0.70
Let P(AB) be the probability of management accountant having both PG and work experience	0.20
Total number of applicants (N)	200

Find

The number of applicants who would have either a PG degree or work experience : N*P(A+B)

Since the events are not mutually exclusive then

$$P(A + B) = P(A) + P(B) – P(AB)$$

Therefore the no of applicants = N * P (A+ B)

$$P (A + B) = P (A) + P (B) – P (AB) = 0.3 + 0.7 – 0.2 = 0.80$$

Hence number of candidates having PG degree and work experience

$$= N * P(A+B)$$

$$= 200 * 0.80 = 160$$

PROBLEM - 4

The probability of a contractor getting a plumbing contract is 2/3 and the probability that he will not get an electrical contract is 5/9. If the probability of getting atleast one contract is 4/5, what is the probability that he will get both ?

Data Given

Let P(A) be the probability of getting plumbing contract — 2/3

Let P(B complement) be the probability of not getting electrical contract — 5/9

Therefore, P(B) be the probability of getting an electrical contract — 1-5/9 = 4/9

Let P(A+B) be the probability of getting atleast one contract — 4/5

Find : The probability of getting both the contracts = P(AB)

Since the events are not mutually exclusive, then

$$P(A+B) = P(A) + P(B) - P(AB)$$

Therefore $P(AB) = P(A) + P(B) - P(A+B)$

$$4/5 = 2/3 + 4/9 - P(AB)$$

(ie.) $P(AB) = 2/3 + 4/9 - 4/5$

$$= 0.66 + 0.44 - 0.80$$

$$= 1.10 - 0.80 = 0.30$$

Probability of getting both the contracts = 0.30

PROBLEM - 5

- **A Chartered Accountant applies for a job in two firms X and Y. He estimates that the probability of getting a job in firm X is 0.7 and being rejected at Y is 0.5 and the probability of atleast one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firms?**

Hint : P(A compliment +B compliment) =1- P(AB)

Data Given

Let P(A) be the probability of getting selected in Firm X — 0.70

Let P(B complement) be the probability of not getting selected — 0.50

Therefore P(B) be the probability of getting selected in Firm

= 1 – P(B complement) — 1-0.50=0.50

Let P(A compliment+ B complement) be the probability of his application being rejected — 0.60

Therefore P(AB) be the probability of getting selected in both the firms

= 1 – P(A compliment+B complement) — 1-0.60=0.40

Find : Probability of getting selected in one of the firms : P (A + B)

Since the events are not mutually exclusive, then

$$P(A+B) = P(A) + P(B) - P(AB) = 0.70 + 0.50 - 0.40 = 0.80$$

80 % he will be selected in one of the firms

Statistical Independence

- When two events A and B are independent then the occurrence of A is not influenced by occurrence of event B.
- When there are more than two events, statistical independence rule says that the occurrence of a particular event is not influenced by the occurrence of the remaining events.
- There are three types of probability under statistical independence given as follows:
  - Joint Probability
  - Marginal Probability
  - Conditional Probability

Two events are independent *if and only if*

$P(A \mid B) = P(A)$ or $P(B \mid A) = P(B)$

Multiplication Rule or Joint Probability for Independent Events

- If two events are independent the probability of happening of event A and event B together is the product of their individual probability:

$$P(A \cap B) = P(A) * P(B)$$

Marginal Probability

- It is the probability of one event that precludes any information about the other event.
- In other words we can say that marginal probability is the unconditional probability P(A) regardless of whether event B did or did not occur.
- Thus the marginal probability of event A is written as P(A) and the marginal probability of event B is P(B)

Conditional Probability

- It refers to the probability of happening event A, given that event B has already occurred. Conditional probability is denoted as P(A|B).

- The probability of A given that B has already happened is given as follows:

$$P(A \mid B) = P(A)$$

- Similarly the probability of B, given that A has already occurred is given as follows:

$$P(B \mid A) = P(B)$$

Statistical Dependence

- Two events A and B are said to be statistically dependent when the probability of event A is dependent on or influenced by the happening of the event B.
- Like independent events, we have three types of probabilities for dependent events:
  - Joint Probability
  - Marginal Probability
  - Conditional Probability

Marginal Probability

- When events are statistically dependent, marginal probabilities are given by the summation of the entire joint probability in which simple event happens.

Conditional Probability

When events are statistically dependent then conditional probability is given by the following expression:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Where
P (A and B) is the joint probability of event A and B
P (A) is the marginal probability of event A
P (B) is the marginal probability of event B

Similarly the conditional probability of P(B|A) is:

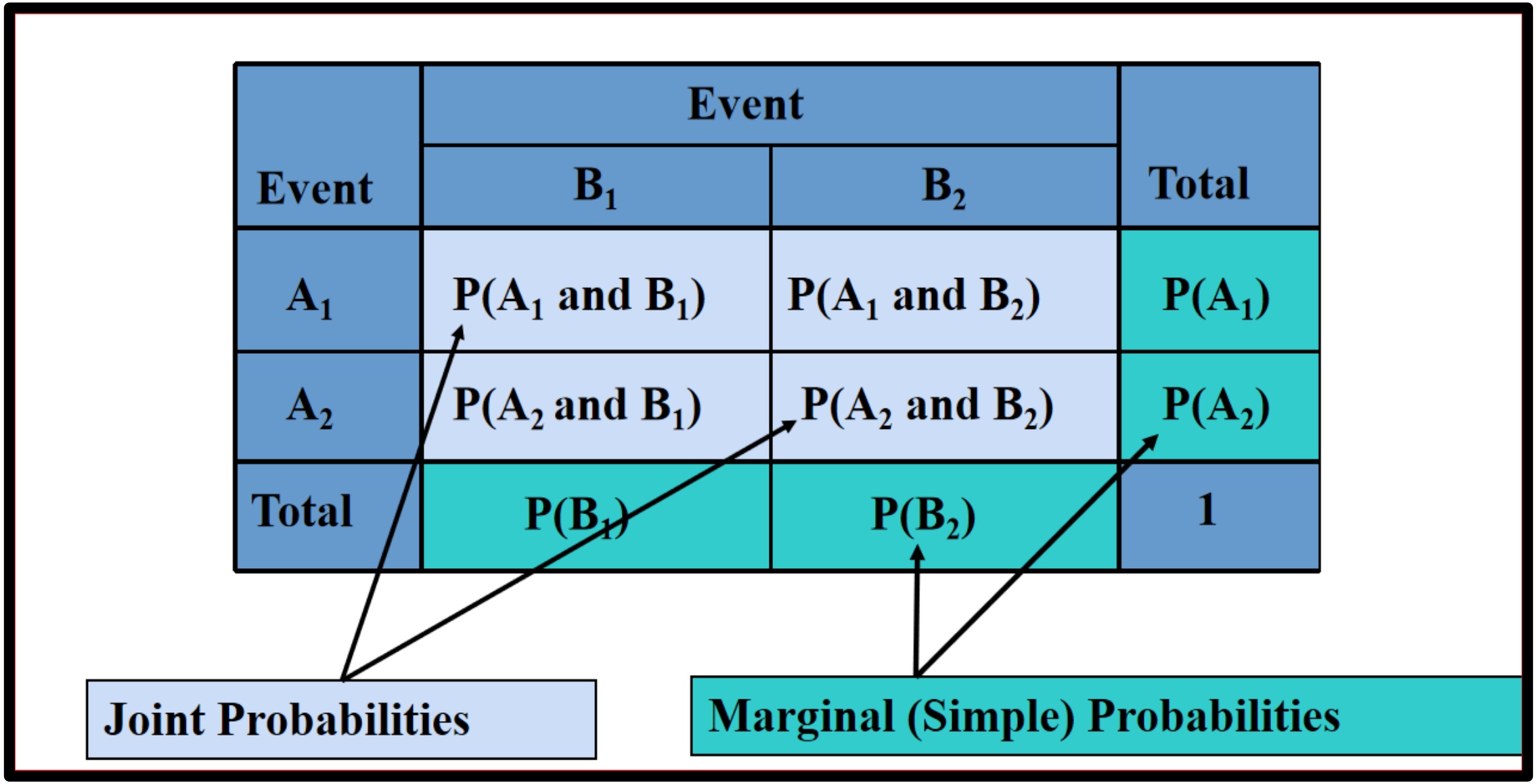
$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Multiplication Rule for Dependent Events or Joint Probability

- If two events are dependent on each other the probability of happening both (joint probability) is the probability of one event multiplied by the probability of other given that the first event has already occurred.

$$P(A \cap B) = P(B \mid A) * P(A)$$

- The term P(B|A) is called the conditional probability of occurring event B given that A has already occurred.


Event
Event
B1
B2
Total
A1
P(A1 and B1)
P(A1 and B2)
P(A1)
A2
P(A2 and B1)
P(A2 and B2)
P(A2)
Total
P(B1)
P(B2)
1
Joint Probabilities
Marginal (Simple) Probabilities

Computing Conditional Probabilities

- A conditional probability is the probability of one event, given that another event has occurred:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

→ The conditional probability of A given that B has occurred.

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

→ The conditional probability of B given that A has occurred.

Where P(A and B) = joint probability of A and B

P(A) = marginal or simple probability of A

P(B) = marginal or simple probability of B

Example

Why are some candidates qualifying GRE others not? One probable reason is whether or not the candidates took coaching. The relationship between qualifying for GRE and coaching was examined and the following information provided:

		B1	B2
		Qualified GRE	Not Qualified GRE
A1	Coaching	0.44	0.16
A2	Self Prepared	0.10	0.30

1. Compute all the joint probabilities
2. Compute the marginal probability
3. What is the probability that a candidate will qualify for GRE given that the candidate has taken coaching?

Solution

Let us denote the events as follows:

A_1 = The candidate took coaching

A_2 = The candidate did not take coaching

B_1 = Qualified GRE

B_2 = Not Qualified GRE

Thus,

$P(A_1 \text{ and } B_1) = 0.44$

$P(A_2 \text{ and } B_1) = 0.10$

$P(A_1 \text{ and } B_2) = 0.16$

$P(A_2 \text{ and } B_2) = 0.30$

The marginal probabilitiesare calculated by adding across rows or down columns.

It is called marginal probabilities because they are calculated in the margins of the table.

The joint probabilities can be used to compute marginal probabilities.

	Qualified GRE	Not Qualified GRE	Totals
Coaching	$P(A_1 \text{ and } B_1) = 0.44$	$P(A_1 \text{ and } B_2) = 0.16$	$P(A_1) = 0.60$
Self Prepared	$P(A_2 \text{ and } B_1) = 0.10$	$P(A_2 \text{ and } B_2) = 0.30$	$P(A_2) = 0.40$
Totals	$P(B_1) = 0.54$	$P(B_2) = 0.46$	1.00

These marginal probabilities shows that 0.54% of all the candidates qualified for GRE and 0.46 % of all the candidates do not qualified for GRE.

Conditional Probability

- The marginal probability that a candidate took coaching is 0.60, which is made of two joint probabilities.
- This probability says, on an average, for every 100 candidates, 60 took coaching. Of these 60, on an average, 44 qualified for GRE. Thus conditional probability is 0.44/0.60 =0.73.
- Thus, the probability that a candidate will qualify for GRE given that he has taken coaching is 0.73.

	Qualified GRE	Not Qualified GRE	Totals
Coaching	$P(A_1 \text{ and } B_1) = 0.44$	$P(A_1 \text{ and } B_2) = 0.16$	$P(A_1) = 0.60$
Self Prepared	$P(A_2 \text{ and } B_1) = 0.10$	$P(A_2 \text{ and } B_2) = 0.30$	$P(A_2) = 0.40$
Totals	$P(B_1) = 0.54$	$P(B_2) = 0.46$	1.00

Contingency Tables and Probabilities

- **Contingency Tables**
  - ✓ Each cell in the contingency table represents a frequency.

Age Group	Brand Name		
	Under Armour	Nike	Adidas
Under 35 years	174	132	90
35 years and older	54	72	78

  - ✓ In the above table, 174 customers under the age of 35 purchased an Under Armour product.
  - ✓ 54 customers at least 35 years old purchased an Under Armour product.

Contingency Tables and Probabilities

- The contingency table may be used to calculate probabilities using relative frequency.
  - ✓ Note: Abbreviated labels have been used in place of the class names in the table.

Age Group	Brand Name			Total
	B_1	B_2	B_3	
A	174	132	90	396
A^c	54	72	78	204
Total	228	204	168	600

  - ✓ First obtain the row and column totals.
  - ✓ Sample size is equal to the total of the row totals or column totals. In this case, $n = 600$.

Contingency Tables and Probabilities

- **Joint Probability Table**
  - ✓ The joint probability is determined by dividing each cell frequency by the grand total.

Age Group	Brand Name B_1	B_2	B_3	Total
A	0.29	0.22	0.15	0.66
A^c	0.09	0.12	0.13	0.34
Total	0.38	0.34	0.28	1.00

Joint Probabilities

Marginal Probabilities

  - For example, the probability that a randomly selected person is under 35 years of age and makes an Under Armour purchase is:

$$P(A \cap B_1) = \frac{174}{600} = 0.29$$

Conceptual Review
Probability

LO : 1 : *Describe fundamenatal probability concepts*

- *An experiment is a process that leads to one of several possible outcomes.*
- *A sample space of an experiment contains all possible outcomes of the experiment , denoted by S .*
- *An event is any subset of outcomes of an experiment,called a simple event, if it contains a single outcome.*
- *Events are exhaustive if all possible outcomes of an experiment belong to the events*
- *Events are mutually exclusive if they do not share any common outcome of an experiment*
- *A probability is a numerical value that measures the likelyhood that an event occurs.It assumes a value between 0 (impossible event) & 1 (definite event).*
- *A **subjective probability** is calculated by drawing on personal & subjective judgement.An **emperical probability** is calculated as a relative frequency of occurence.A **classical probability** is logical analysis rather than on observation or personal judgement*

Conceptual Review
Probability

LO2 Apply the Rules of Probability :

*The **complement rule** states that the probability of the complement of an event can be found by subtracting the probability of the event from one .* *P(A complement) = 1- P(A)*

*The probability that atleast one of 2 events occurs is calculated by using the **addtion rule.***

***P(A+B) = P(A) + P(B) - P(AB)** for not mutually exclusive events ,P(A) is an unconditional probability.*

***P(A+B) = P(A)+P(B)** for mutually exclusive events since P(AB) =0*

The probability that A occurs given that B has already occured ,denoted by P(A/B) is a conditional probability.

LO3 Distinguish between independent & Dependent events

A conditional probability is computed as **P(A/B) =** $\frac{P(AB)}{P(B)}$ *(or)* **P(B/A) =** $\frac{P(AB)}{P(A)}$

*For Dependent events, Multiplication rule is , P(AB)=P(B)*P(A/B)*

For independent events , Multiplication rule is P(AB) = P(A) P(B)*

Conceptual Review
Probability

LO4 ***Calculate & Interpret probabilities from a contingency table***

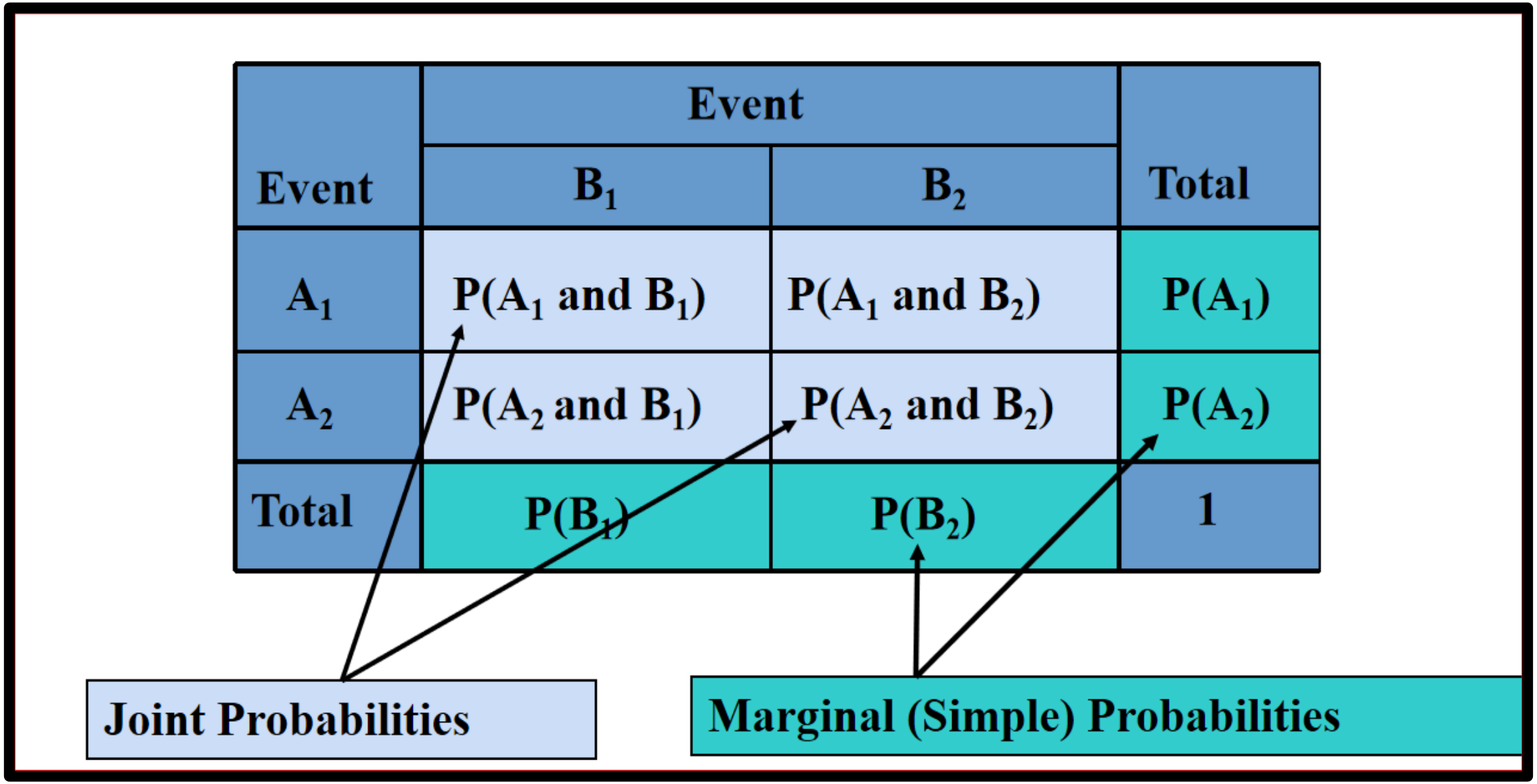
*A **contingency table** generally shows frequencies for 2 qualitative (categorical) variables, x and y, where each cell represents a mutually exclusive combination of x-y values.Emperical probabilities are easily calculated as the relative frequency of the occurence of the event.*

LO5 ***Apply the total probability rule***

The total probability rule is

$$P(A) = P(AB1) + P(AB2) + \ldots\ldots\ldots + P(ABn) \text{ - Independent event}$$

$$P(A) = P(A/B1)*P(B1) + P(A/B2)*P(B2) + \ldots\ldots\ldots + P(A/Bn)*P(Bn) \text{ -Dependent event}$$


Event
Event
B1
B2
Total
A1
P(A1 and B1)
P(A1 and B2)
P(A1)
A2
P(A2 and B1)
P(A2 and B2)
P(A2)
Total
P(B1)
P(B2)
1
Joint Probabilities
Marginal (Simple) Probabilities

PROBLEM - 6

Suppose a company hires both MBA's and non MBA's and after a period of employment some are promoted and some are not. Table given below are the proportion of company managers among the said class. Note : If A and B are said to be independent, then P(A/B) = P(A) and P(B/A) = P(B)

Status	A (MBA)	A complement (non MBA)	Total
Promoted (B)	0.42 AB	0.18	0.60 B
Not Promoted ($\bar{B}$)	0.28	0.12	0.40
Total	0.70 A	0.30	1.00

Calculate (i) P (A/B) and P (B/A) (ii) Find whether A and B are independent events.

P(A/B) = P(AB) / P(B)
= 0.42/0.6
= 0.70

P(B/A) = P(AB) / P(A)
= 0.42/ 0.7 = 0.6

If events are independent, P(A/B) = P(A). Here P(A) = 0.70 & P(A/B) = 0.70
P(B/A) = P(B). Here P(B) = 0.60 &
P(B/A).=0.60

Since the condition is satisfied A & B are independent

PROBLEM - 7

On the basis of information given by a reliable source, there was a plan for sudden raid by the government authority on some shops. The shops that were decided to be raided were 55 out of 100 in that area. The following is the estimated classification based on the information given below.

Goods	Big Shop (C)	Small shop (D)	Total
Non smuggled (A)	25 AC	20	45 AD
Smuggled (B)	35	20 BD	55
Total	60	40	100

Find P (C/A), P (A), P (B/D), P (A/D), P (B)),P(C)

P(A)= 45/100=0.45

P(D)= 40/100=0.40

P(B)= 55/100=0.55

P(C)= 60/100=0.60

P(C/A) = P(AC) / P(A)

P(C/A) = 0.25/0.45 =0.56

P(AC)= 25/100 =0.25

P(B/D) = P(BD) / P(D)

P(B/D) = 0.20/0.40 =0.5

P(BD)= 20/100 =0.20

P(A/D) = P(AD) / P(D)

P(A/D) = 0.20/0.40 =0.5

P(AD)= 20/100 =0.20

PROBLEM - 8

The probability that A can solve a problem in business statistics is 4 / 5. The probability of B solving the problem is 2 / 3. The probability of C solving the problem is 3 / 7. If all of them are trying independently, find the probability that the problem will be solved ?

Data Given

Let P(A) be the probability of A solving the problem — 4/5

Let P(B) be the probability of B solving the problem — 2/3

Let P(C) be the probability of C solving the problem — 3/7

Find : The probability that all of them trying independently to solve the problem.: P(A + B + C)

Method 1 : **P(Problem will be solved) = P(A solving the problem or B solving the problem or C solving the problem**

P (A+B+C) = 1- P(problem will not be solved) Since the events are independent

= 1- P(A compliment*B compliment*C compliment)

= 1 – (1/5 * 1/3 * 4/7) = 1 – (4/105) = 101/105 = 0.96

Method 2: **P(A+B+C) = P(A) + P(B) + P(C) – P(AB) – P(BC) – P(CA) + P(ABC)**

= 4/5 + 2/3 + 3/7 – (4/5*2/3) - (2/3*3/7) – (4/5*3/7) + (4/5*2/3*3/7)

= 0.80 + 0.66 + 0.42 – 0.53 – 0.28 - 0.34 + 0.23 = 0.96

PROBLEM - 9

The odds against student X solving a business statistics problem are 8 to 6 and the odds in favour of Y solving the same are 14 to 16. What is the probability that the problem will be solved if both try independently ?

Data Given

Student	Solving	Not solving	Total
X	6	8	14
Y	14	16	30

Let P(A) be the probability of X solving the problem 6/14=0.42

Let P(B) be the probability of Y solving the problem 14/30=0.46

Let P(AB) be the probability of both solving the problem = P(A)P(B) 0.42*0.46=0.19

Find : The probability that the problem will be solved if both of them try independently (ie) Either A solving the Problem or B solving the problem : P(A+B)

Since the events are not mutually exclusive, then

$$P(A+B) = P(A) + P(B) - P(AB)$$
$$= 0.42 + 0.46 - (0.42*0.46)$$
$$= 0.88 - 0.19$$
$$= 0.69$$

Alternatively

P(A+B) = 1- P(None of them solving the problem

. = 1- (P (A compliment)*P(B compliment))

= 1- (0.58*0.54) = 1- 0.3132

= 0.6868 say 0.69

PROBLEM - 10

- The personnel department of a company has the following analysis of its 200 engineering graduates.

Age	Bachelor's Degree (A)	Masters Degree (B)	Total
Under 30 (C)	90 AC	10	100
30 – 40 (D)	20	30	50
Over 40 (E)	40	10 BE	50
Total	150	50	200

If an engineer is selected at random from the company, find

(i) the probability that he has only a bachelor's degree. P(A) 150/200 = 0.75

(ii) the probability that he has a masters degree given that he is over 40. P(B/E) P(BE) /P(E) (10/200) /(50/200)= 0.2

(iii) the probability that he is under 30 given that he has only a bachelor's degree. P(C/A) P(CA) /P(A)

(90/200)/(150/200)= 0.6

PROBLEM - 11

In a locality out of 5000 people residing, 1200 are above 30 years of age, and 3000 are females. Out of 1200 who are above 30, 200 are females. Suppose after a person is chosen, you are told that the person is female. What is the probability that she is above 30 years of age ?

Data Given

Given Data

Age	Male (C)	Female (D)	Total
Below 30 (A)	1000	2800	3800
Above 30 (B)	1000	200	1200
Total	2000	3000	5000

FIND

The probability that a female is chosen who is above 30 years of age. P(B/D)

Let P(B/D) be the probability that chosen person is above 30 years given that she is a Female.

$$P(B/D) = P(DB)/P(D) = (200/5000)/(3000/5000)$$

$$= 0.04/0.60 = 0.066$$

BAYES THEOREM

The Total Probability Rule and Bayes' Theorem

- **Bayes' Theorem**
  - ✓ A procedure for updating probabilities based on new information.
    - **Prior probability** is the original (unconditional) probability (e.g., $P(B)$).
    - **Posterior probability** is the updated (conditional) probability (e.g., $P(B \mid A)$).

The Total Probability Rule and Bayes' Theorem

- **Bayes' Theorem**
  - ✓ Given a set of prior probabilities for an event and some new information, the rule for updating the probability of the event is called **Bayes' theorem**.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A \cap B) + P(A \cap B^c)}$$

or

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A \mid B)P(B) + P(A \mid B^c)P(B^c)}$$

Bayes Theorem

- Determining the probability of an event given certain sample information, Bayes theorem is applied

- Probability before revision by Bayes rule is called prior probability, because they are determined before the sample information is taken into account.

- A probability which has undergone revision in the light of sample information through Bayes rule is called posterior probability since it represents a probability calculated after the information is taken into account.

- Posterior probability is also called the revised probability because they are obtained by revising the prior probability in the light of the additional information gained.

- Posterior probability is always conditional probability. The conditional event being the sample information. Thus a prior probability which is an unconditional probability becomes a posterior probability which is a conditional probability by Bayes rule.

- The revision of the old probability in the light of the additional information supplied by the experiment and or past records is of extreme help to business managers in arriving at a valid decision in the phase of uncertainity.

Bayes Theorem

- Let $B_1, B_2, B_3 \ldots\ldots\ldots\ldots B_n$ are mutually exclusive and collectively exhaustive events with prior probability $P(B_1)$, $P(B_2)$,$P(B_3)$……..$P(B_n)$. Let A is another event such that $P(A) \neq 0$ and the conditional probability $P(A/B_1)$, $P(A/B_2)$……..$P(A/B_n)$ are known then

$$\mathbf{P\ (\ Bi/A) = (\ P(\ A/Bi)\ P(Bi)\ )\ /\ \sum P(Bi)\ P(A/Bi)}$$

Bayes Theorem – Example

	Event		Prior Probability	
A Priori Probability	S1	CPA Score ≥750	P(S1 ≥750) = 0.25	These are your estimates
(subjective probability)	S2	CPA Score < 750	P(S2≤750) = 0.75	Events are mutually exclus collectively exhaustive
			1.00	(All possible outcomes)

Event : New Information from Survey)			
Took Preparatory Course	P(PC) | ≥750	0.60	Using Relative frequency approach
Not taken Preparatory course	P(PC) | < 750	0.31	Not list of all conditional probabilities.Not add to 1

Event	**P(Si)** Prior Probabilities	Event	**P(PC | Si)** Conditional Probability	**Joint Probability**	P(Si | PC) Posterior Probability
P(S1 ≥ 750)	0.25	P(PC) | ≥750	0.60		
P(S2< 750)	0.75	P(PC) | < 750	0.31		
			P(PC) =		1.00

We Know

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Joint Probability P(AB) = P(A | B) * P(B)

Conclusion : With new information , the probability we pass CPA given that we take Preparatory course is 0.39, Better than the original of 0.25

P(S1 | PC) = 0.39

Example 1

- In XYZ company, three machines, *M*1, *M*2, and *M*3, make 30%, 45%, and 25%, respectively of the products. The past experience shows that 2%, 3%, and 2% of the products made by each machine respectively are defective. Suppose that a final product is randomly chosen what is the probability that it is defective? If the product was chosen randomly and found to be defective, what is the probability that it was made by machine *M*3?

Data Given

Let us assume the following events:

D: the product is defective
M_1: the product is made by machine M1
M_2: the product is made by machine M2
M_3: the product is made by machine M3

Now we can utilize the given information to arrive at the conditional probabilities as follows:

P(D|M1)= 0.02
P(D|M2) = 0.03
P(D|M3) = 0.02

Solution

Let us assume the following events:

D: the product is defective

M_1: the product is made by machine M1

M_2: the product is made by machine M2

M_3: the product is made by machine M3

P(M1) =0.30

P(M2) =0.45

P(M3) =0.25

Now we can utilize the given information to arrive at the conditional probabilities as follows:

P(D|M1)= 0.02

P(D|M2) = 0.03

P(D|M3) = 0.02

Now the joint probabilities are computed as follows:

P(M1) P(D|M1)= (0.30)(0.02) =0.006

P(M2) P(D|M2) = (0.45)(0.03) = 0.0315

P(M3) P(D|M3) = (0.25)(0.02) =0.005

Now the total probability is:

P(D) = P(M1) P(D|M1) + P(M2) P(D|M2)+ P(M3) P(D|M3)

= 0.006+0.0315+0.005

= 0.0245

Thus if a final is randomly selected the probability is 2.45% that it is defective.

Now in order to find out the probability that it was made by M3 Bayes' rule can be applied as follows:

$$P(M_3 \mid D) = \frac{P(D \mid M_3)P(M_3)}{P(D)} = \frac{0.005}{0.0245} = 0.2048$$

Thus there is 20.48% chance that it was made by M3.

	Prior Probability	Conditional Probability	Joint Probability	Posterior Probability
M1	0.30	P(D|M1)= 0.02	0.006	0.244897959
M2	0.45	P(D|M2)=0.03	0.0135	0.551020408
M3	0.25	P(D|M3)=0.02	0.005	0.204081633
			0.0245	1

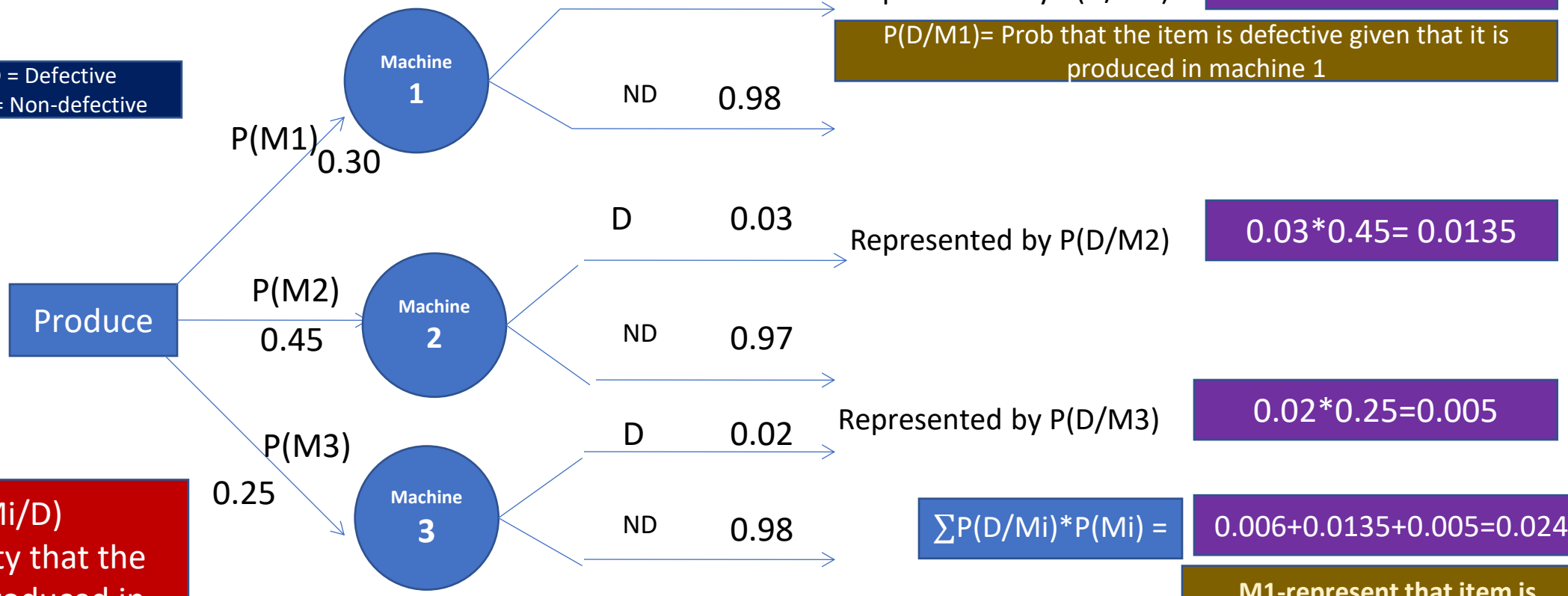
Decision Tree

- Decision tree diagram is a graphical representation of various alternatives and the sequence of events in a decision problem .In constructing a decision tree there are certain conditions to be followed .
- The tree is constructed starting from left and moving towards right , The square box represents a decision point at which the available strategies are considered , The circle represents the chance node or event , The various states of nature ,outcomes emanate from this chance event .At each decision or chance node there can be one or more branches represented by a straight line .
- The decision tree shows the structure of the decision problem for finding the Probability using Bayes theorem .

BAYES SOLUTION – Decision Tree approach

P(A/Bi)*P(Bi) / ∑P(A/Bi)*P(Bi) = P(Bi/A)

D = Defective
ND = Non-defective



P(D/Mi)P(Mi)

0.02*0.3= 0.006

P(D/M1)= Prob that the item is defective given that it is produced in machine 1

0.03*0.45= 0.0135

0.02*0.25=0.005

∑P(D/Mi)*P(Mi) =

0.006+0.0135+0.005=0.0245

P(Mi/D)
Probability that the item is produced in M/C "i" given that it is defective

Prior	Find	Working	Posterior
P(M1)=0.30	P(M1/D)	0.006/0.0245	0.24489
P(M2)=0.45	P(M2/D)	0.0135/0.0245	0.5510
P(M3)=0.25	P(M3/D)	0.005/0.0245	0.20408
1.00			1.000

M1-represent that item is produced in Machine 1

M2 represent that item is produced in machine 2

M3 represent that item is produced in machine 3

BAYES THEOREM - PROBLEM - 2

- A restaurant is experiencing dissatisfaction among its customers. Its analysis states that there are 3 factors responsible : 1. Food quality 2. Service quality 3. Interior decoration. By conducting an analysis it assesses the probability of dissatisfaction with these three factors are 0.4, 0.35 and 0.25 respectively.

- By conducting a survey among customers, it also evaluates the probability of a customer going away with dissatisfaction on account of these factors are 0.6, 0.5, and 0.2 respectively. With this information, the restaurant wants to know that if a customer is dissatisfied, what are the probabilities that it is so due to food, service ,interior decoration?

BAYES THEOREM – PROBLEM1

- Let

P(B1) – Probability that the customer is dissatisfied due to food quality- 0.40

P(B2) - Probability that the customer is dissatisfied due to sevice quality- 0.35

P(B3) – Probability that the customer is dissatisfied due to Interiors - 0.25

This is called Prior probability (Before conducting the expt we know the value of probability)

From the sample survey

Let " A " represent Dissatisfaction

P(A/B1) – Probability that the customer is dissatisfied given that it is due to food quality – 0.6

P(A/B2) – Probability that the customer is dissatisfied given that it is due to service quality – 0.5

P(A/B3) – Probability that the customer is dissatisfied given that it is due to interiors - 0.2

Through Bayes Theorem,we can revise the value of probability

$P(B1/A) = (P(A/B1) * P(B1)) / \sum P(A/Bi)*P(Bi)$

$P(B2/A) = (P(A/B2) * P(B2)) / \sum P(A/Bi)*P(Bi)$

$P(B3/A) = (P(A/B3) * P(B3)) / \sum P(A/Bi)*P(Bi)$

P(B1/A) –given that the customer is disatisfied on account of Food quality

P(B2/A) – given that the customer is dissatisfied on account of service quality

P(B3/A) –given that the customer is dissatisfied on account of interiors

After conducting the sample survey ,we revise the value of probability.It is called Posterior probability

BAYES SOLUTION - 2

P(A/Bi)*P(Bi)

P(A/Bi)*P(Bi) / ∑P(A/Bi)*P(Bi) = P(Bi/A)

A = Dissatisfied
S = Satisfied

Dissatisfaction Due to

- P(B1) 0.40 → Food Quality
  - A 0.60 → Represented by P(A/B1) 0.6*0.4= 0.24
  - S 0.40
- P(B2) 0.35 → Service Quality
  - A 0.50 → Represented by P(A/B2) 0.5*0.35= 0.175
  - S 0.50
- P(B3) 0.25 → Interior Decoration
  - A 0.20 → Represented by P(A/B3) 0.25*0.20=0.05
  - S 0.80

P(A/B1)- Prob.that the customer is dissatisfied given that it is due to food quality = 0.60

P(A/B2)- Prob.that the customer is dissatisfied given that it is due to service quality =0.50

∑P(A/Bi)*P(Bi) = 0.24+0.175+0.05 = 0.465

B1-represent Dissatisfaction due to food quality

B2-represent Dissatisfaction due to service quality

B3-represent Dissatisfaction due to Interiors

Prior prob	Find	Working	Posterior prob
P(B1)= 0.40	P(B1/A)=	0.24/0.465	0.516
P(B2) = 0.35	P(B2/A)=	0.175/0.465	0.376
P(B3)= 0.25	P(B3/A)=	0.05/0.465	0.108
1.00			1.000

BAYES Theorem - PROBLEM - 3

- In a bolt factory, machine X,Y,Z manufacturers 25%, 35% and 40% of the total output and 5%, 4% and 2% are defective bolts. A bolt is drawn at random What is the prob that it was manufactured by machine X,Y,Z
- Let

P(B1) – Probability that the bolt is produced in machine X- 0.25

P(B2) - Probability that the bolt is produced in machine Y- 0.35

P(B3) – Probability that the bolt is produced in machine Z - 0.40

This is called Prior probability (Before conducting the expt we know the value of . probability)

From the sample survey

Let

" A " represent the Defective bolt

P(A/B1) – Probability that the produced bolt is defective given that it is produced in X – 0.05

P(A/B2) – Probability that the produced bolt is defective given that it is produced in Y – 0.04

P(A/B3) – Probability that the produced bolt is defective given that it is produced in Z - 0.02

BAYES SOLUTION - 2

P(A/Bi)*P(Bi) / ∑P(A/Bi)*P(Bi) = P(Bi/A)

A = Defective
ND = Non-defective

Produce

P(M1) 0.30 → Machine X
- D 0.02 → Represented by P(D/M1)
- ND 0.98

P(M2) 0.45 → Machine Y
- D 0.03 → Represented by P(D/M2)
- ND 0.96

P(M3) 0.25 → Machine Z
- D 0.02 → Represented by P(D/M3)
- ND 0.98

P(A/Bi)P(Bi)

0.02*0.3= 0.006

P(A/B1)= Prob that the item is defective given that it is produced in machine X

0.03*0.45= 0.0135

0.02*0.25=0.005

∑P(A/Bi)*P(Bi) =

0.006+0.0135+0.005=0.0245

P(Bi/A)
Probability that the item is produced in M/C "i" given that it is defective

Prior	Find	Working	Posterior
P(M1)=0.30	P(M1/D)	0.006/0.0245	0.24489
P(M2)=0.45	P(M2/D)	0.0135/0.0245	0.5510
P(M3)=0.25	P(M3/D)	0.005/0.0245	0.2040
1.00			1.000

B1-represent that item is produced in Machine X

B2 represent that item is produced in machine Y

B3 represent that item is produced in machine Z

Summary

- **Discussed basic probability concepts**.
  - Sample spaces and events, contingency tables, simple probability, and joint probability
- **Examined basic probability rules.**
  - General addition rule, addition rule for mutually exclusive events, rule for collectively exhaustive events.
- **Defined conditional probability.**
  - Statistical independence, marginal probability, decision trees, and the multiplication rule
- **Discussed Bayes' theorem.**

Application of combination theory

For solving probability problems

- # What is Factorial?
  - In many problems product of consecutive integers are needed. For instance the product of 6, 5, 4, 3, 2, 1 is called 6 factorial and denoted by *6!*.
  - In general, *n!* is called n-factorial which denotes the product of first *n* natural numbers.
  - Few Examples
    - 0! = 1
    - 1! = 1
    - 2! = 2x1 = 2
    - 3! = 3x2x1 =6
    - 4! = 4x3x2x1= 24 and so on.

Counting Rules

- The **Combination Formula**
  - ✓ The number of ways to choose *x* objects from a total of *n* objects, where the order in which the *x* objects are listed *does not matter*, is referred to as a **combination** and is calculated as:

$$ {}_nC_x = \binom{n}{x} = \frac{n!}{(n-x)!x!} $$

Counting Rules

- The **Permutation Formula**
  - ✓ The number of ways to choose *x* objects from a total of *n* objects, where the order in which the *x* objects is listed *does matter*, is referred to as a **permutation** and is calculated as:

$$ {}_nP_x = \frac{n!}{(n-x)!} $$

Permutation

- It refers to the different ways in which a number of objects can be arranged in a definite order.
- For example there are two products X and Y. These products can be arranged in two different ways i.e., XY and YX. The two arrangements are called permutations.

- The permutation of 'n' different objects taken r at a time is given by the following expression:

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

where

n = number of different objects given

r = number of different objects taken at a time out of different objects given n!

$^{n}P_{r}$ = denote the number of permutations of n different objects taken r at a time.

$$^{6}P_{3} = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120$$

Thus, if there are total 6 objects and if 3 are taken at a time then they can be arranged in 120 ways.

(b)

$$^{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = 6720$$

Thus, if there are total 8 objects and if 5 are taken at a time then they can be arranged in 6720 ways.

Combinations

- It refers to the number of arrangements which can be made from a group of items irrespective of the order.

- The combination of 'n' different objects taken r at a time is given by the following expression:

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

(a)

$$^{12}C_{5} = \frac{n!}{r!\,(n-r)!} = \frac{12!}{5!\,7!} = \frac{12\times11\times10\times9\times8\times7!}{5\times4\times3\times2\times1\times7!} = \frac{95040}{120} = 792$$

(b)

$$^{4}C_{2} = \frac{n!}{r!\,(n-r)!} = \frac{4!}{2!\,2!} = \frac{4\times3\times2!}{2\times1\times2!} = \frac{12}{2} = 6$$

Factorials

- **Example :**

  Suppose you want to select 2 students out of 3 students (say X, Y and Z) for a dinner meeting with Warren Buffet in how many ways can the selection be made

No of ways of selecting 2 students are XY, XZ, YZ. Total no of ways = 3.

Mathematical way of representing the same is

$$nCr = n!/r!(n-r)!$$

It means no of ways of selecting r successes in n trials

In the above example n = 3 and r = 2.

3C2 = 3!/2!(3-2)! =(3*2*1) /(2*1)*1 = 6/2 = 3

0 ! = 1
1! = 1
2! = 2*1
3! =3*2*1
4! =4*3*2*1

Alternatively
3c2 = (3*2)/(1*2) = 3

- **Suppose, n is 4 instead of 3, then the formula would be**

4C2 = 4!/2!(4-2)!=4*3*2*1/2*1*2*1=24/4 = 6

2 students out of 4 from W, X, Y and Z.

WX, WY, WZ, XY, XZ, YZ (no of ways =6)

Alternatively
4c2= (4*3) / (1*2) = 6

PROBLEM - 1

- **A bag contains 6 white, 4 red and 10 black balls. 2 balls are drawn at random. Find the probability that both of them are black ?**

Data Given

No. of White balls	6
No .of Red balls	4
No. of Black balls	10
Total number of balls	20

Solution :

Probability that both are black = 45 / 190

= 0.236

10C2= (10*9)/(1*2) = 45

20C2 = (20*19)/(1*2) =190

2 balls are drawn. What is the Probability that both are black

10 C2 represents no of ways of drawing 2 black balls out of 10 black balls

20 C2 represent no of ways of drawing 2 balls out of 20 balls

Find : The probability that both are black. = 10C2 / 20C2

PROBLEM - 2

- **A bag contains 8 white balls, 4 black balls. 5 balls are drawn at random. What is the probability that 2 of them are black and 3 are white ?**

Data Given

Number of white balls	8
Number of black balls	4
Total number of balls	12

Solution :

Probability of drawing 2 black balls and 3 white balls = 4C2 * 8C3 / 12C5

= (6*56) / 792 = 0.424

4C2= (4*3)/(1*2) =6

8C3 = (8*7*6)/(1*2*3)= 56

12C5 = (12*11*10*9*8) / (1*2*3*4*5) = 792

Find : Probability of getting 2 black and 3 white balls when drawing 5 balls at random.

Probability (drawing 2 black balls and 3 white balls) = (4C2 * 8C3) / 12C5

PROBLEM - 3

- **A box contains 3 red, 7 white balls. 1 ball is drawn at random and in its place a ball of the other colour is put in the box. Now a ball is drawn from that box. Find the probability that it is red.**

Data Given :

Number of red balls	3
Number of white balls	7
Total number of balls	10
Probability of drawing red ball is	P(R)
Probability of drawing white ball is	P(W)

Find : When one ball is drawn at random and in its place a ball of the other colour is put in the box, find the probability that when a ball is drawn, it is red.

SOLUTION - 3

When ball drawn is	Probability	Replaced Ball	No. of balls
P (R)	3/10	White	R = 2 ; W = 8 ; T = 10
P (W)	7/10	Red	R = 4 ; W = 6 ; T = 10

P(R)

2/10

4/10

Required Probability = 3/10 * 2/10 + 7/10 * 4/10

= 6/100 + 28/100

= 34/100

= 0.34

Let P(R) be the probability of drawing a red ball
Let P(W) be the probability of drawing a white ball

PROBLEM - 4

- **Two bags contain each 5 black and 4 white balls. A person draws a ball at random from the first bag and puts it in the second bag. Now he draws a ball from the second bag. What is the probability that it is white ?**

Data Given :

Number of Black balls is 5

Number of White balls is 4

Total number of balls is 9

Probability of drawing black ball is P(B)

Probability of drawing white ball is P(W)

Find : When a ball is drawn at random from the first bag and put in the second, find the probability that the ball drawn from the second bag is white?

SOLUTION - 4

No. of balls in the first bag	When ball drawn is from the first bag	Probability	After putting the ball in the second bag	Probability of drawing a white ball = P(W)
No of Black balls = 5	Black ,then P(B)	5/9	No of balck balls=5+1 No of White balls= 4 Total balls = 10	4/10
No of White balls = 4 Total no of balls=9	White,then P(W)	4/9	No of White balls =4+1 No of Black balls = % Total balls = 10	5/10

Required probability is = 5/9 * 4/10 + 4/9 * 5/10

= 20/90 + 20/90

= 40/90

= 4/9

Conceptual Review
Probability

LO : 1 : *Describe fundamenatal probability concepts*

- *An experiment is a process that leads to one of several possible outcomes.*
- *A sample space of an experiment contains all possible outcomes of the experiment , denoted by S .*
- *An event is any subset of outcomes of an experiment,called a simple event, if it contains a single outcome.*
- *Events are exhaustive if all possible outcomes of an experiment belong to the events*
- *Events are mutually exclusive if they do not share any common outcome of an experiment*
- *A probability is a numerical value that measures the likelyhood that an event occurs.It assumes a value between 0 (impossible event) & 1 (definite event).*
- *A **subjective probability** is calculated by drawing on personal & subjective judgement.An **emperical probability** is calculated as a relative frequency of occurence.A **classical probability** is logical analysis rather than on observation or personal judgement*

Conceptual Review
Probability

LO2 Apply the Rules of Probability :

*The **complement rule** states that the probability of the complement of an event can be found by subtracting the probability of the event from one .* *P(A complement) = 1- P(A)*

*The probability that atleast one of 2 events occurs is calculated by using the **addtion rule.***

***P(A+B) = P(A) + P(B) - P(AB)** for not mutually exclusive events ,P(A) is an unconditional probability.*

***P(A+B) = P(A)+P(B)** for mutually exclusive events since P(AB) =0*

The probability that A occurs given that B has already occured ,denoted by P(A/B) is a conditional probability.

LO3 Distinguish between independent & Dependent events

A conditional probability is computed as **P(A/B) =** $\frac{P(AB)}{P(B)}$ *(or)* **P(B/A) =** $\frac{P(AB)}{P(A)}$

*For Dependent events, Multiplication rule is , P(AB)=P(B)*P(A/B)*

For independent events , Multiplication rule is P(AB) = P(A) P(B)*

Conceptual Review
Probability

LO4 ***Calculate & Interpret probabilities from a contingency table***

*A **contingency table** generally shows frequencies for 2 qualitative (categorical) variables, x and y, where each cell represents a mutually exclusive combination of x-y values.Emperical probabilities are easily calculated as the relative frequency of the occurence of the event.*

LO5 ***Apply the total probability rule***

The total probability rule is

$$P(A) = P(AB1) + P(AB2) + \ldots\ldots\ldots + P(ABn) \text{ - Independent event}$$

$$P(A) = P(A/B1)*P(B1) + P(A/B2)*P(B2) + \ldots\ldots\ldots + P(A/Bn)*P(Bn) \text{ -Dependent event}$$

Conceptual Review
Probability

LO6 **Apply Bayes theorem**

Bayes Thorem provides a procedure for updating probabilities based on new information.

Let P(B) be the prior probability and P(B/A) be the posterior probability based on new information provided by A . Then

$$P(Bi/A) = \frac{P(A/Bi)\,P(Bi)}{\sum P(Bi)\,P(A/Bi)}$$

LO7 **Use a counting rule to calculate the probability of an event**

Shortcut rules for counting include the **factorial** , the **combination ,** and the **permutation** formulas.The combination and permutation formulas apply to 2 groups of predetermined size. We apply the combination formula when the order of the arrangement does not order.