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Hypothesis Testing

Learning Objectives

You will learn:

- The basic principles of hypothesis testing
- How to use hypothesis testing to test a mean or proportion
- The assumptions of each hypothesis-testing procedure, how to evaluate them, and the consequences if they are seriously violated
- How to avoid the pitfalls involved in hypothesis testing
- Ethical issues involved in hypothesis testing

Notations

	Population	Sample
Average or Mean	μ (mu)	$\ddot{X}$ (X bar)
Variance	σ^2	s^2
Standard deviation	σ	s
Proportion	Π	p1

Population parameter and Sample statistics

What is called Population parameter and sample statistics ?

Population Parameter	Sample Statistics
Statistical measures representing the 1.Population mean (μ) 2.Population Proportion (p) 3.Population Standard deviation (σ) are called parameter.	Statistical measure representing 1.Sample mean (ẍ) 2. Sample proportion (p_1) 3. Sample std. Deviation (s) are called sample statistics.

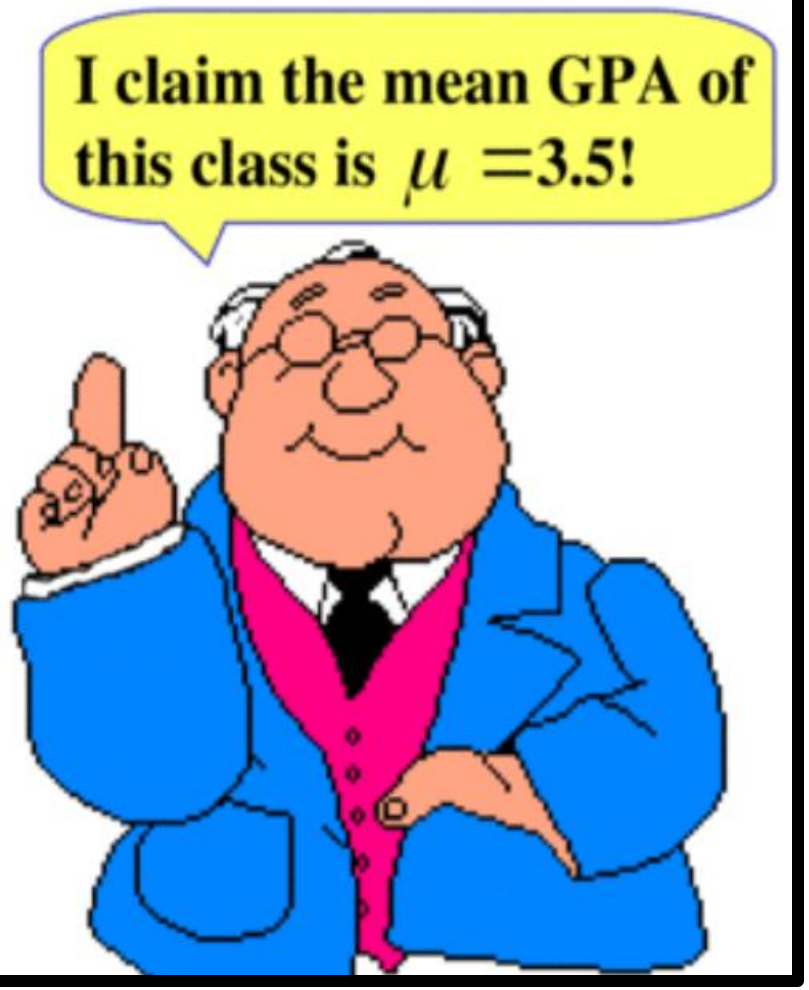
A Hypothesis is a claim or assumption about a population parameter

In hypothesis testing population parameter is tested based on sample statistics since population parameter is considered to be unknown

When the data is Normal, if Population parameter is tested based on sample statistics. then it is called Parametric testing .

What is a Hypothesis?

- A Hypothesis is a Claim (Assertion) about the Population Parameter
  - Examples of parameters are population mean (μ) or proportion (P)
  - The parameter must be identified before analysis



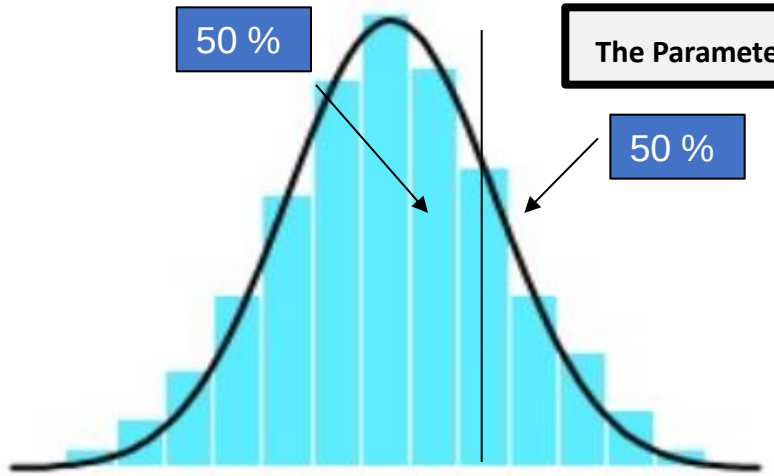
Any data collected from the natural process will be Normal.
If the data is Normal , Mean = Median = Mode

- The commonest and the most useful continuous distribution
- A symmetrical probability distribution where most results are located in the middle and few are spread on both sides
- It has shape of a bell
- Can entirely be described by its mean and standard deviation

Mean = 154

Median = 154

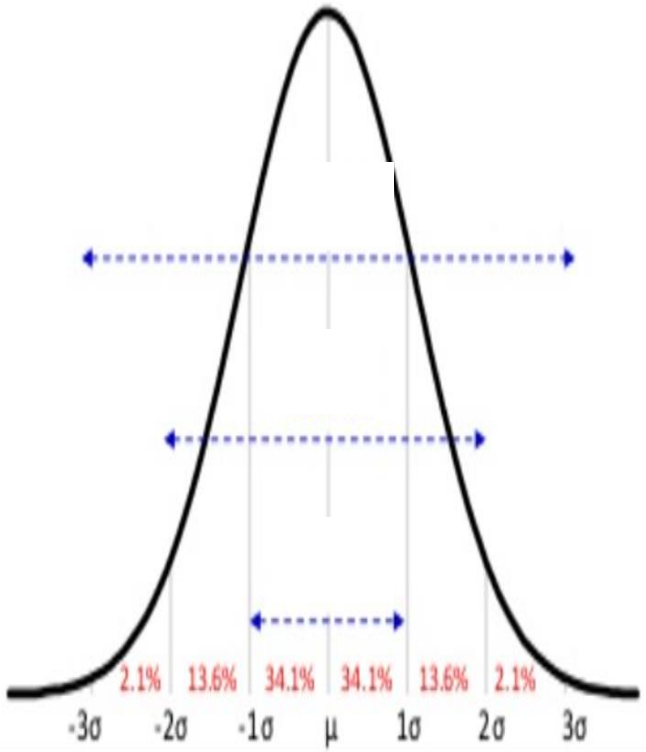
Mode = 154



The Parameter of Normal Distribution is μ & σ

Empirical Rule:

- For any normally distributed data:
  - ____ of the data fall within 1 standard deviation of the mean.
  - ____ of the data fall within 2 standard deviations of the mean.
  - ____ % of the data fall within 3 standard deviations of the mean.



Z table

How to use Z table:

The values inside the given table represent the areas under the standard normal curve for values between 0 and the relative z-score. For example, to determine the area under the curve between 0 and 2.36, look in the intersecting cell for the row labeled 2.30 and the column labeled 0.06. The area under the curve is .4909.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0.004	0.008	0.012	0.016	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.091	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.148	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.17	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.195	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.219	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.258	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.291	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.334	0.3365	0.3389
1	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.377	0.379	0.381	0.383
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.398	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.437	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.475	0.4756	0.4761	0.4767
2	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817

The Hypothesis

- A hypothesis is a claim (assumption) about a population parameter:
  - population mean

    Example: The mean monthly cell phone bill of this city is $\mu = \$52$

  - population proportion

    Example: The proportion of adults in this city with cell phones is $\pi = .68$

The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

> Example: The average number of TV sets in homes is equal to three ($H_o : \mu = 3$

- Is always about a population parameter, not about a sample statistic



$H_o : \mu = 3$

$H_o : \bar{X} = 3$

- It is a tentative conclusion or assumption or declaration or statement which may or may not be true.
- It is usually worded as “No significant difference between population parameter and sample statistics”.

The Null Hypothesis, H_0

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- It refers to the status quo
- **Always contains "=" , "≤" or "≥" sign**

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis
  - e.g., The mean number of TV sets in U.S. homes is not equal to 3 ($H_1: \mu \neq 3$)
- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- Always contains "≠" , "<" or ">" sign
- May or may not be proven
- Is generally the hypothesis that the researcher is trying to prove

Formulating the right Hypothesis for 1 set of data Quiz

No of sample : 1

If you are trying to show that Average pay in a company is greater than Rs 20000	H_0	**μ <= 20,000**
	H_1	**μ > 20,000**
If we were to show that the proportion of defects in a sample is less than 25 %	H_0	p >= 0.25
	H_1	p < 0.25
If we were to show that the variance of a population is same as 100	H_0	**$\sigma^2 = 100$**
	H_1	**$\sigma^2 \neq 100$**
If we were to show that Data is Normal	H_0	Data is normal
	H_1	Data is not normal

Formulating the right Hypothesis for more than 1 set of data points

Quiz **No of sample > 1**

If we were to show that Average transaction time is same in the 2 shifts

H_o $\mu_1 = \mu_2$

H_1 $\mu_1 \neq \mu_2$

If we are comparing the proportion of more than 2 populations and we want to show that they are different

H_o

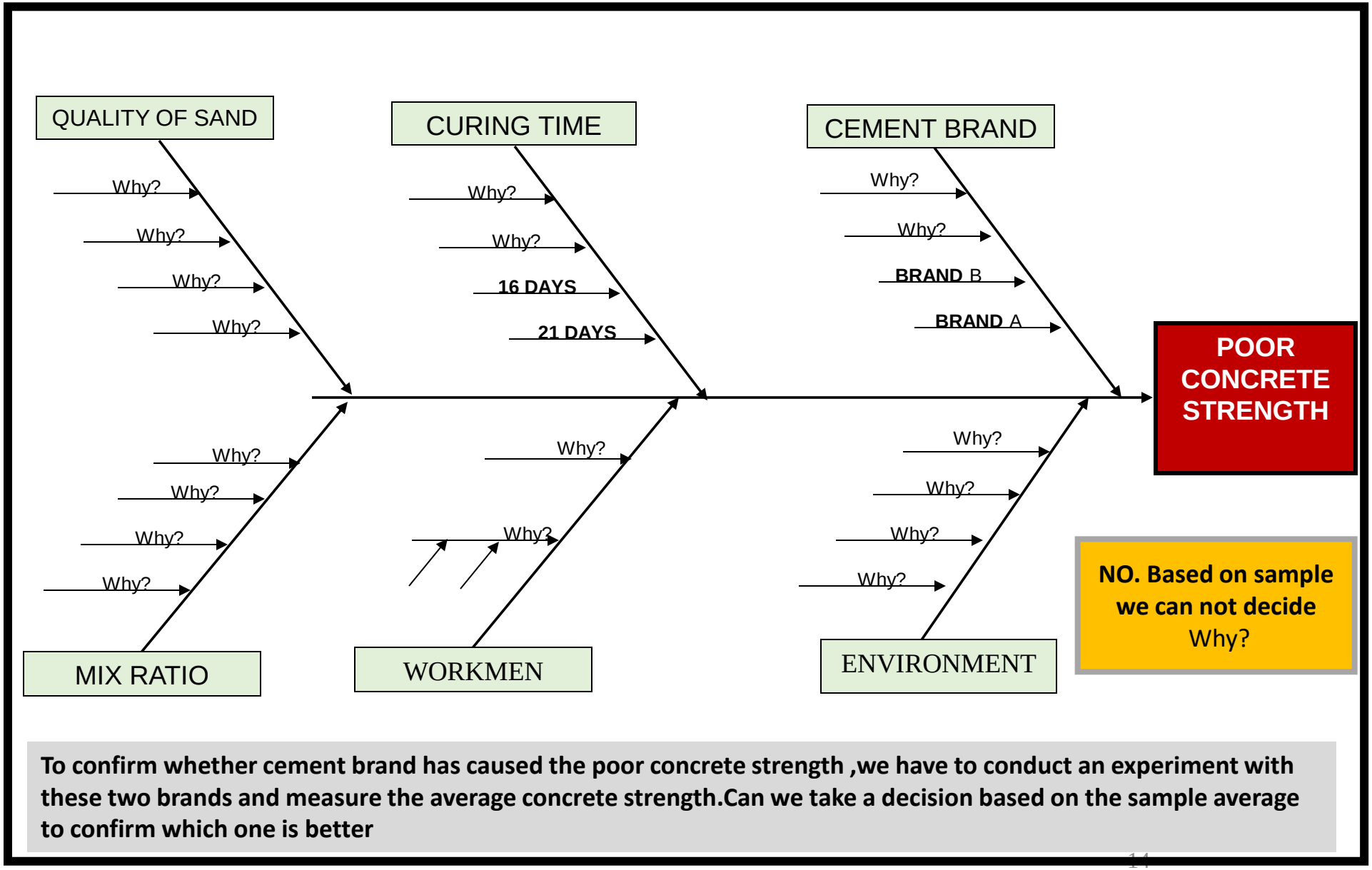
H_1 Atleast 1 proportion is different from others

If we were to show that the data was random and there were trends in data

H_o There are no trends

H_1 There are trends

CAUSE AND EFFECT DIAGRAM



To confirm whether cement brand has caused the poor concrete strength ,we have to conduct an experiment with these two brands and measure the average concrete strength.Can we take a decision based on the sample average to confirm which one is better

Why we can not decide based on sample results

Maruti limited claims that their Alto vehicle will give 19 kms/litre . Suppose there are 10 maruti cars in the population and their kms per litre is as follows

20 ,15, 18, 19, 18, 22, 16, 19, 18, 25		Total =190 Population average (μ) =19

To test maruti's claim of 19 km/litre , 4 agencies collected the data from 3 customers from the population.

AGENCY	SAMPLE VALUE	SAMPLE TOTAL	SAMPLE AVERAGE	REMARKS	CONCLUSION
1	18,15,16	49	16.33	< μ	NOT VALID
2	15,16,19	50	16.67	< μ	NOT VALID
3	19,18,20	57	19	= μ	VALID
4	22,20,25	67	22.33	> μ	VALID

Why we can not decide based on sample results

In some sample $\ddot{x}$ is greater than μ and in some other sample $\ddot{x}$ is less than μ,hence we can not accept or reject the Maruti's claim based on sample.. Decision to be taken only after statistical testing, which is called hypothesis testing

Hypothesis

What is hypothesis ?

A theory or idea which is not proved yet.

This can be classified as

1. **Null hypothesis – Statement of no difference**
2. **Alternate hypothesis – Statement of difference**

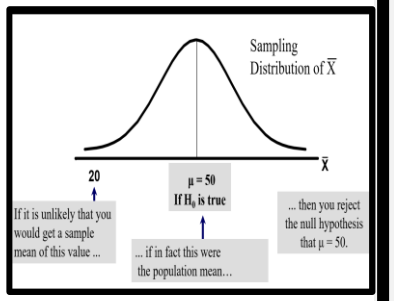
Example : **Medicine B for headache that is newly developed by a pharmaceutical company has 30 minutes longer effect than existing medicine A.**

H0 : Medicine A and B have the same effect

H1 : Medicine B has 30 minutes longer effect than medicine A

Develop the hypothesis for population and make statistical decisions by determining the acceptance of the hypothesis using sample data

The Hypothesis Testing Process



- **Claim:** The population mean age is 50.

  $H_0: \mu = 50$, $H_1: \mu \neq 50$

  Suppose the sample mean age was X bar = 20.

- This is significantly lower than the claimed mean population age of 50.

  - *If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis .*

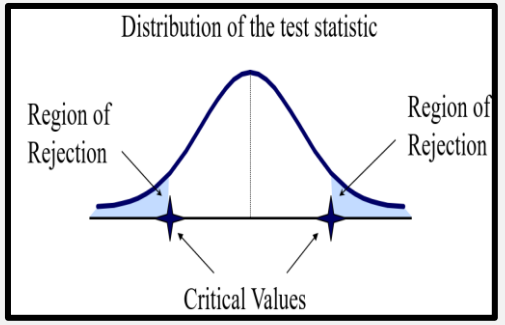
- In other words, getting a sample mean of 20 is so unlikely if the

  population mean was 50, you conclude that the population mean .

. . must not be 50.

The Test Statistic and Critical Values

- If the sample mean is close to the assumed population mean, the null hypothesis is not rejected.
- If the sample mean is far from the assumed population mean, the null hypothesis is rejected.
- How far is "far enough" to reject H_0?



- The critical value of a test statistic creates a "line in the sand" for decision making.

Decision errors

- There is always a possibility of errors in the Decisions we make.
- Two types of errors can result from a hypothesis test.
- They are

Type I error : **A type I error occurs when you reject a null hypothesis when it is true .** The probability of committing a Type I error is called the significance level, denoted by alpha. It is also called Producer's risk

1- alpha is called the confidence level

Type II error: **It occurs when you accept (fails to reject) the null hypothesis when it is false .** The probability of committing a Type II error is called Beta .It is also called consumer's risk. The probability of not committing a Type II error is called the **Power of the Test (1- beta)**

DECISION ERRORS

• Type I error

When good lot is rejected due to sampling then it is called TYPE I Error.
(ie) Rejecting a Null Hypothesis when it is true.
It is also called as Producer's risk .It is denoted by α ,which is called Level of Significance

10,000 apples received .
The customer can go for 100% inspection or sampling inspection.
The following situation the customer will face

What is Level of Significance :
It is the percentage of error in the conclusion arrived with the help of sample.

The standard levels of α are 5% and 1%. **Default percentage is 5%**

1- α is called Confidence level

When α = 5% , it indicates 5% of the time good lot is rejected

Here 1- α = 95 % , which indicates 95% of the time good lot is accepted

SITUATION 1	INPUT	100 % INSPECTION	SAMPLING INSPECTION
	ACCEPTANCE CRITERIA	1% rejection permitted (i.e) in 10,000 apples 100 damages permitted	Take 1000 nos,10 damages permitted
	INSPECTION RESULT	62 damages noticed	12 damages noticed
	CONCLUSION	Accept the lot	Reject the lot

DECISION ERRORS

- **Type II error**

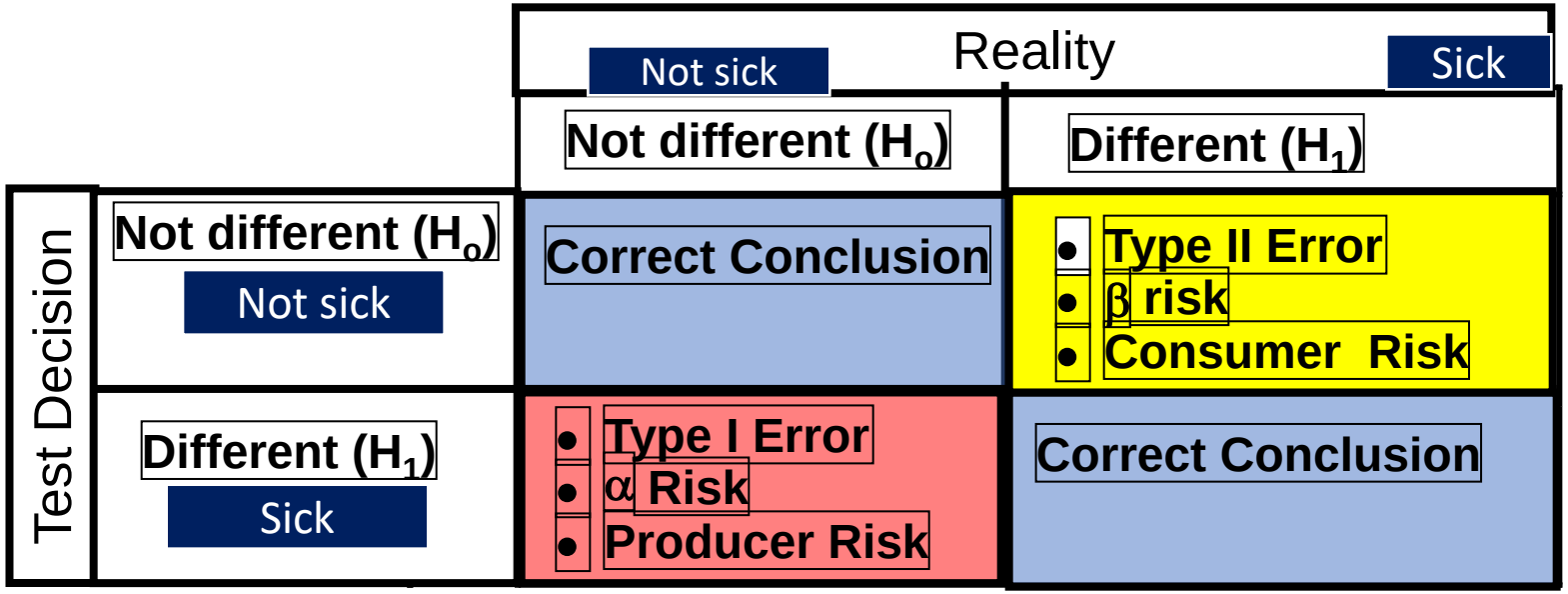
10,000 apples received .
The customer can go for 100% inspection or sampling inspection.
The following situation the customer will face

INPUT	100 % INSPECTION	SAMPLING INSPECTION
ACCEPTANCE CRITERIA	1% rejection permitted (i.e) in 10,000 apples 100 damages permitted	Take 1000 nos,10 damages permitted
INSPECTION RESULT	130 damages noticed	9 damages noticed ;
CONCLUSION	Reject the lot	Accept the lot

When bad lot is accepted due to sampling then it is called TYPE II Error.
(ie) Accepting a Null Hypothesis when it is False
It is also called as consumer's risk .It is denoted by β . The std. level of β is 10 %
(ie) 10 % of the time the bad lot is accepted.
1- β is called the POWER OF THE TEST
(ie) 90% of the time the bad lot is rejected

How confident do you want to be that you have made the right decision?

A person does not feel well and checks into a hospital for tests.



Ho: Patient is not sick

H1: Patient is sick

Error Impact

Type I Error = Treating a patient who is not sick

Type II Error = Not treating a sick patient

DECISION ERRORS

Two Errors in Hypothesis Testing

TRUTH	DECISION	
	ACCEPT NULL H_0	**REJECT NULL H_0**
H_0 NULL IS TRUE GOOD LOT	CORRECT DECISION 1- α (e.g. 0.95) Confidence level	**TYPE 1 Error** α (e.g 0.05) Significance Level Producers risk
H_0 NULL IS FALSE BAD LOT	**TYPE 2 ERROR** β (0.10) Consumers risk	CORRECT DECISION 1- β (0.90) power of the test

TYPE 1 error: Punishing an Innocent person.
TYPE 2 error Guilty person set free.

BASIC CONCEPTS-SAMPLING QUIZ

1.The probability of failing to reject a null hypothesis when it is actually true refers to:

a. Type 1 error
b. Type 2 error
c. Power of a test
d. None of these.

d. None of these

2.The probability of rejecting a null hypothesis when it is actually false is known as :

a. Type 1 error
b. Type 2 error
c. power of a test
d. Significance level

c. Power of a test
Type 2 error occurs when a null hypothesis is accepted when it is actually false.

3.The probability of not rejecting a null hypothesis when it is actually false is referred to as :

a. Type 1 error
b. Type 2 error
c. power of a test
d. significance level

b. Type 2 error

Level of Significance, α

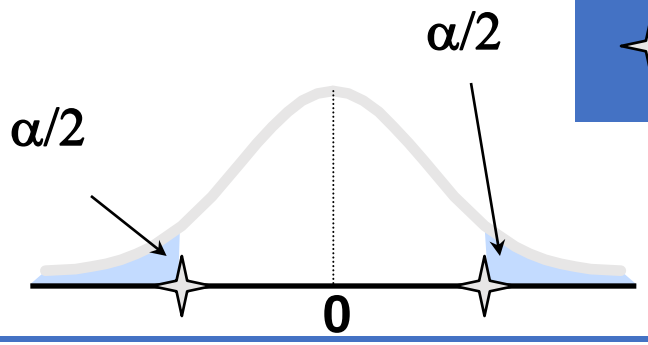
Claim: The population mean age is 50.

2 sided test

$H_0: \mu = 50$

$H_1: \mu \neq 50$

Two-tail test



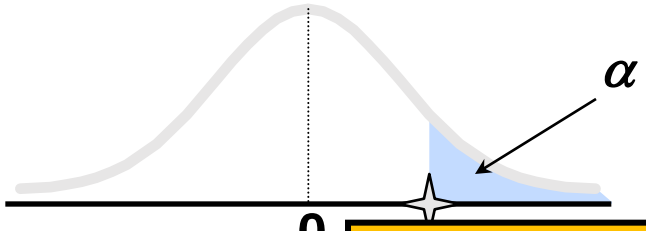
Represents critical value

Rejection region is shaded

$H_0: \mu \leq 50$

$H_1: \mu > 50$

Upper-tail test



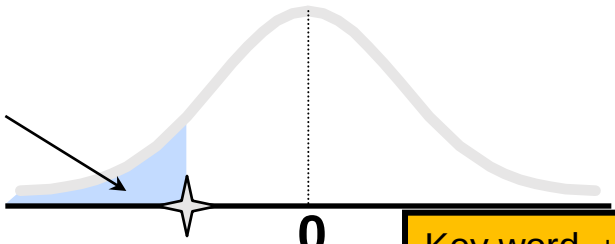
one sided test

Key word : greater than,More than,atleast

$H_0: \mu \geq 50$

$H_1: \mu < 50$

Lower-tail test



one sided test

Key word : less than,lower,reduced

Identify the key word in the problem for one sided test :

- ***Increase/Decrease***
- ***More than/Less than***
- ***Better***
- ***Atleast***
- ***improvement***

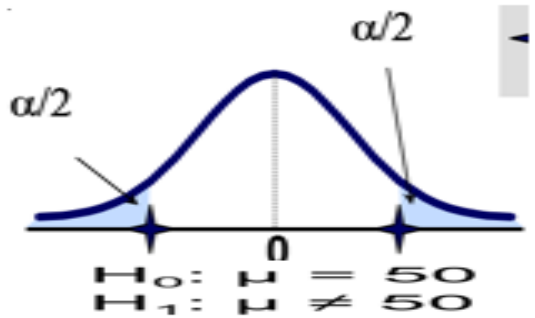
Note :

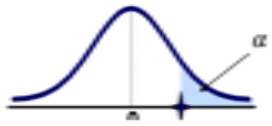
Decision can go wrong , if you select 2 tail test instead of one tail test

Z statistics WITH THE LEVEL OF SIGNIFICANCE α

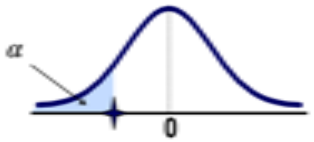
Alpha	Two tailed Test (1-α/2)	One tailed Test (1-α) RHS	LHS
1%	2.58	2.33	-2.33
5%	1.96	1.645	-1.645
10%	1.645	1.28	-1.28

1- 0.025 = 0.975





$H_0: \mu \leq 50$
$H_1: \mu > 50$



$H_0: \mu \geq 50$
$H_1: \mu < 50$

Key word →

Increase / Decrease
More than / Less than
Atleast , Better,
reduce, effective

Quiz

For a test with alpha =0.02 and beta = 0.05 , the following is true,

A Confidence level = 98% , Power= 95%

B Confidence level = 95% , Power= 95%

C Confidence level = 98% , Power= 98%

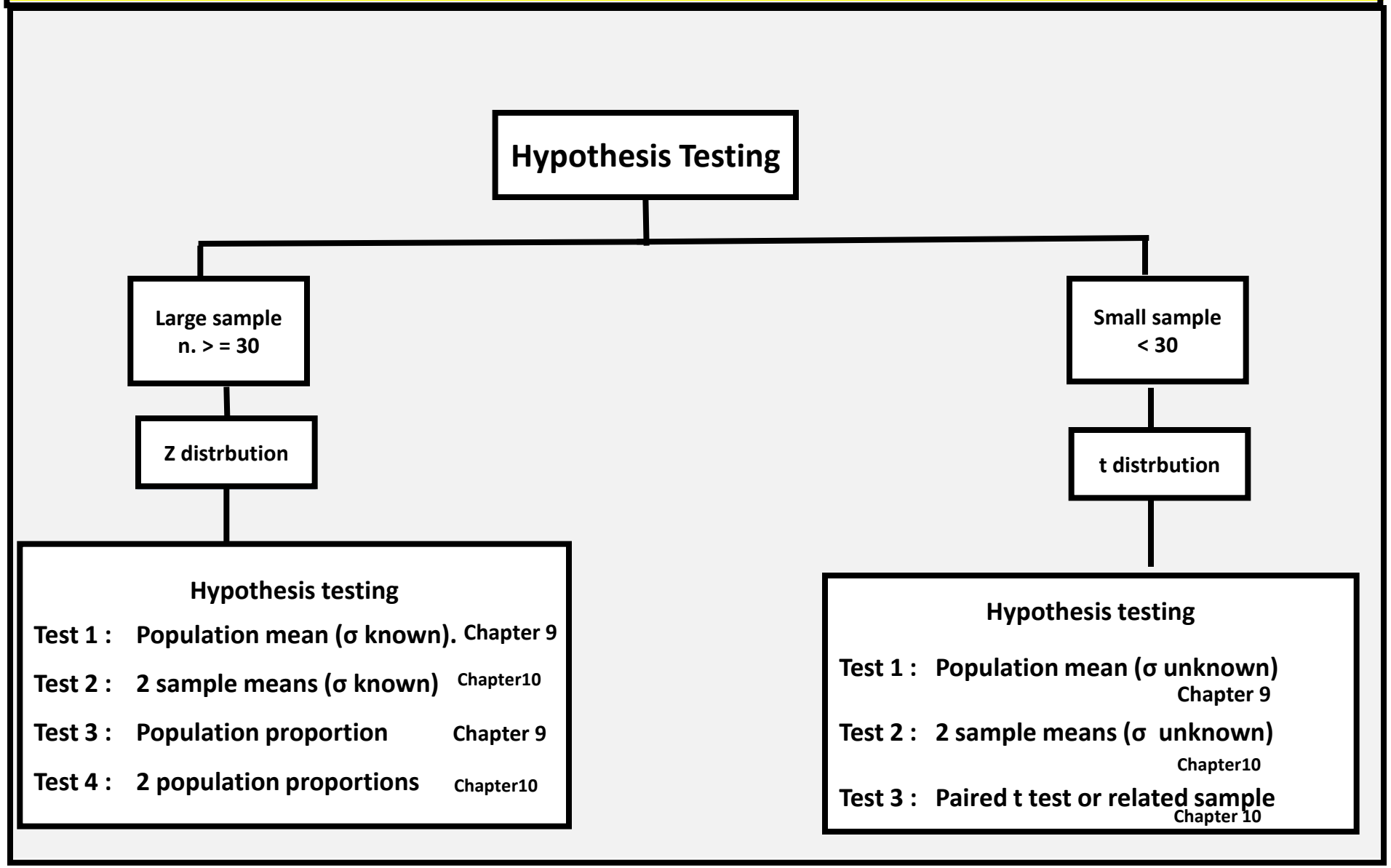
D Confidence level = 95% , Power= 98%.

A Confidence level = 98% , Power= 95%

Confidence Level = 1-α

Power of the test = 1- β

Hypothesis Testing



P value

Hypothesis	P value	Decision	Conclusion
Ho : $\mu 1 = \mu 2$ H1: $\mu 1 \neq \mu 2$	P= 0.023	Reject H0	There is a significant difference between averages
Ho :Data is Normal H1:Data is not Normal	P= 0.652	Accept H0	Data is Normal
Ho : $\sigma_1^2 = \sigma_2^2$ Ho : $\sigma_1^2 \neq \sigma_2^2$	P= 0.000	Reject H0	Variances are not equal

A small or low value of p means a small probability that H0 is true
A large p value means a large probability that H0 is true

$\alpha = 0.05$

Based on p value Decision to be taken.

$P <= \alpha$ Reject H_o

$P > \alpha$, Accept H_o

Hypothesis decision taken based on p value

No of sample	Hypothesis		P value	Decision	Conclusion
one	Average pay in company A > 20000	H0 :μ=20000 H1: μ >20000	P= 0.035	Reject H0	Average pay in company A > 20000
Two	MRF tyre averge life is > ceat tyre's avg.life	H0 : μ1=μ2 H1: μ1 > μ2	p ≥α	Accept H0	Not valid

To reject MRF claim what should be the value of p

If one set of data is collected(Maruti) then No of sample is one

P value is the probability of making a Type 1 error

Decision : If p value is > 0.05 (α), Accept Null Hypothesis (H0)

Hypothesis Testing

No of sample- one

Output Data

Measurable(continuous)

Parameter to be tested Average

Data to be given

Population average	μ
Population SD	σ
Sample size	n
Sample average	$\bar{X}$

If LOS is not given Assume 5 %

Step : 1	Find the difference	$\bar{X} - \mu$
Step : 2	Find the standard error	$\sigma / \sqrt{n}$
Step : 3	Test Statistics	Z = Difference / SE
Step : 4	Decision	Accept H0 if calculated value of z is > table value of z
Step : 5	Conclusion	

Z distribution Large sampling n>= 30

t distribution Small sampling n<30

Test 1 : Population mean (σ known).

Test 2 : Population mean (σ unknown)

$H_0: \mu = 50$ $H_1: \mu \neq 50$ Two-tail test

$H_0: \mu \leq 50$ $H_1: \mu > 50$ Upper-tail test

$H_0: \mu \geq 50$ $H_1: \mu < 50$ Lower-tail test

2 Tail Test

1 Tail Test

EXAMPLE: Maruti claims that his vehicle will give 20km/litre. To test his claim the data collected from 100 maruti customers w.r.t the fuel consumption , to check whether maruti's claim is valid or not.

Countable (Discrete)

Parameter to be tested proportion

Z distrbution

Data to be given

π	=Proportion of responses
No of Trials	
Event	
p =	Sample proportion = $\frac{Event}{Trials}$

Step : 1	Find the difference	$\lvert p - \pi \rvert$
Step : 2	Find the standard error	$\sqrt{\frac{\pi(1-\pi)}{n}}$
Step : 3	Test Statistics	Z = Difference / SE
Step : 4	Decision	Accept H0 if calculated value of z > table value of z
Step : 5	Conclusion	

Hypothesis testing

Test 3 : Population proportion

$H_0: \pi = .08 \quad H_1: \pi \neq .08$

$H_0: \pi \leq 08 \quad H_1: \pi > .08$

- EXAMPLE: Shop keeper claims that 60% of the customers entering the shop do not buy anything. Test whether the shop keeper claim is valid from the samples collected .

Hypothesis Testing – Problem 1

A manufacturer of TV tubes has for many years used a process, gave an average tube life of 4700 hrs and a standard deviation of 1460 hours. A new process is tried to see whether there is an increase in life significantly. A sample of 100 new tubes gave an average life of 5000 hours. Is the new process better than the old process at 1% LOS.

Testing the Population Mean when σ is known. Use Z distribution

=NORM.S.DIST(2.05,1) → 0.979818

Since p is >α ,accept H0

Data Given

No. of samples :		1 (Tv tube)
Population average	μ	4,700 hrs
Population SD	σ	1,460 hrs
Sample size	n	100
Sample average	$\dot{X}$	5,000 hrs
Key word		Better than
Level of significance	α	1%

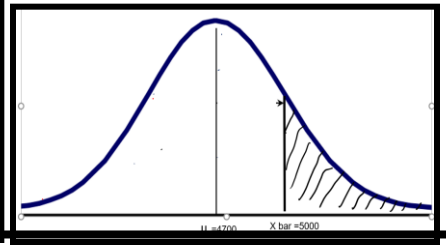
=NORM.S.INV(0.99)

Key word is " Better than, hence " apply one tail test , table value of z is 2.33

Ho :μ = 4700 hrs

H1 :μ > 4700 hrs

Step : 1	Find the difference	$\dot{X} - \mu$	5000 - 4700	300
Step : 2	Find the standard error	σ / √n	1,460 / √100	146
Step : 3	Test Statistics	Z = Difference / SE	Z = 300 / 146 = 2.05	
Step : 4	Decision	Accept H0 since calculated value of z (2.05) is < 2.33		
Step : 5	Conclusion	New process is no way better than the old process		



	5% LOS	1% LOS
One Tail Test	1.64	2.33
Two Tail Test	1.96	2.58

Hypothesis Testing – Problem 1 **Using Excel**

A manufacturer of TV tubes has for many years used a process, gave an average tube life of 4700 hrs and a standard deviation of 1460 hours. A new process is tried to see whether there is an increase in life significantly. A sample of 100 new tubes gave an average life of 5000 hours. Is the new process better than the old process at 1% LOS.

Testing the Population Mean when σ is known. Use Z distribution

	B	C	D
1			
2	**Data Given**		
3			Excel Comman
4	Population Average	4700	
5	population std.deviation	1460	
6	Sample size (n)	100	
7	Sample Average	5000	
8	Difference	300	=C7-C4
9	LOS	1%	
10	One tail test (better)		
11	Critical Z	2.326	=NORM.S.INV(0.99)
12	Standard error	146	=C5/SQRT(C6)
13	Z Test Statistics	2.054795	=C8/C12

Ho :μ = 4700 hrs

H1 :μ > 4700 hrs

Step 1 : Find the difference = $\ddot{X} - \mu$ = 300

Step 2 : Find Std error : $\frac{\sigma}{\sqrt{n}} = 146$

Step 3 : Find $Z\ critical = 2.326$

Step 4 : Find $Z\ test\ statistics =$ 2.054

Step 5 : Decision :
Since calculated value of Z is < table value of Z, accept H0

Step 6 : Conclusion : New Process is no way better than old process

Hypothesis Testing – Problem 2

An insurance agent has claimed that the average age of policy holders who insure through him is less than the average for all agents , which is 30.5 years. A random sample of 100 policy holders who had insured through him gave the following age distribution. Test the agent's claim @ 5 % LOS

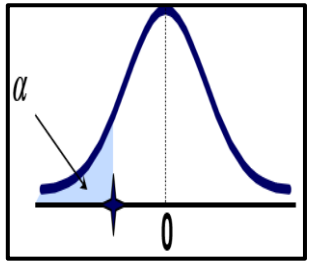
	A	B	C	D	E	F
1	AGE	NO OF PERSONS (f)	MID POINT(x)	fx	f(x-mean)^2	
2	16-20	12	18	216	1399.68	=B2* ((C2-D$11)^2)
3	21-25	22	23	506	740.08	
4	26-30	20	28	560	12.8	
5	31-35	30	33	990	529.2	
6	36-40	16	38	608	1354.24	
7	Total	100		2880	4036	
8		=SUM(B2:B6)		=SUM(D2:D6)	=SUM(E2:E6)	
9						
10						
11			Mean	28.8	=D7/B7	
12			variance	40.76767677	=E7/(B7-1)	
13			s.d	6.384957069	=SQRT(D12)	

Data Given

No. of samples		1 Agent
Population average	μ	30.5 yrs
Sample SD	s	6.38 yrs
Sample size	n	100
Sample average	$\dot{X}$	28.8 yrs
Key word		less than
Level of significance	α	5%

=NORM.S.INV(0.95)

Key word is " Less than, hence " apply one tail test , table value of z is 1.64



Testing the Population Mean , Use Z distribution since n > 30

Ho :μ = 30.5 years

H1 :μ < 30.5 years

Step : 1	Find the difference	$\dot{X} - \mu$	28.8– 35.5	- 1.7
Step : 2	Find the standard error	s / √n	6.38 / √100	0.638
Step : 3	Test Statistics	Z = Difference / SE	Z =- 1.7 / 0.638 = -2.66	
Step : 4	Decision	Reject H0 since calculated value of z (2.66) is > 1.64		
Step : 5	Conclusion	Life insurance agent's claim is valid		

=NORMSDIST(-2.66,1) p= 0.003907

Since p is < 0.05,Reject H0

	5% LOS	1% LOS
One Tail Test	1.64	2.33
Two Tail Test	1.96	2.58

	A	B	C	D	E	F
1	AGE	NO OF PERSONS (f)	MID POINT(x)	fx	f(x-mean)^2	
2	16-20	12	18	216	1399.68	=B2* ((C2-D$11)^2)
3	21-25	22	23	506	740.08	
4	26-30	20	28	560	12.8	
5	31-35	30	33	990	529.2	
6	36-40	16	38	608	1354.24	
7	Total	100		2880	4036	
8		=SUM(B2:B6)		=SUM(D2:D6)	=SUM(E2:E6)	
9						
10						
11			**Mean**	**28.8**	=D7/B7	
12			**variance**	40.76767677	=E7/(B7-1)	
13			**s.d**	6.384957069	=SQRT(D12)	

	A	B	C	D	E	F
1	AGE	NO OF PERSONS (f)	MID POINT(x)	fx	f(x-mean)^2	
2	16-20	12	18	216	1399.68	=B2* ((C2-D$11)^2)
3	21-25	22	23	506	740.08	
4	26-30	20	28	560	12.8	
5	31-35	30	33	990	529.2	
6	36-40	16	38	608	1354.24	
7	Total	100		2880	4036	
8		=SUM(B2:B6)		=SUM(D2:D6)	=SUM(E2:E6)	
9						
10						
11			**Mean**	**28.8**	=D7/B7	
12			**variance**	40.76767677	=E7/(B7-1)	
13			**s.d**	6.384957069	=SQRT(D12)	

	A	B	C	D	E	F
1	AGE	NO OF PERSONS (f)	MID POINT(x)	fx	f(x-mean)^2	
2	16-20	12	18	216	1399.68	=B2* ((C2-D$11)^2)
3	21-25	22	23	506	740.08	
4	26-30	20	28	560	12.8	
5	31-35	30	33	990	529.2	
6	36-40	16	38	608	1354.24	
7	Total	100		2880	4036	
8		=SUM(B2:B6)		=SUM(D2:D6)	=SUM(E2:E6)	
9						
10						
11			**Mean**	**28.8**	=D7/B7	
12			**variance**	40.76767677	=E7/(B7-1)	
13			**s.d**	6.384957069	=SQRT(D12)	

Hypothesis Testing: σ Known

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average more than $52 per month. The company wishes to test this claim. Past company records indicate that the standard deviation is about $10. Suppose a sample is taken with the following results: n = 64, X bar = 53.1 (σ=10 was assumed known from past company records) . Suppose that α = .10 is chosen for this test

Testing the Population Mean when σ known. Use Z distribution

=NORM.S.DIST(0.88,1)

p= 0.81057

Since p is > alpha ,Accept Ho

Ho :μ = $52

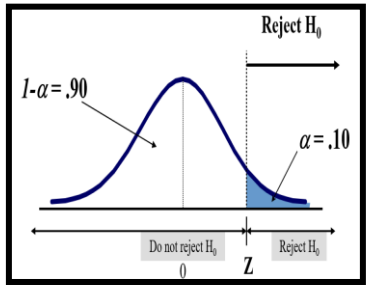
H1 :μ > $52

Data Given

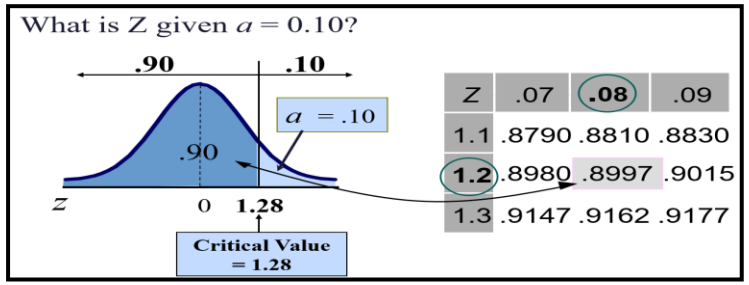
No. of samples :	one(Phone bill)	
Population average	μ	$52
Population SD	σ	$10
Sample size	n	64
Sample average	$\dot{X}$	53.1
Key word	Increase	
Level of significance	α	10%

=NORM.S.INV(0.9)

Key word is " Increase, hence " apply one tail test , table value of z is 1.28



Step : 1	Find the difference	$\dot{X} - \mu$	53.1 -52	1.1
Step : 2	Find the standard error	σ / √n	10 / √64	1.25
Step : 3	Test Statistics	Z = Difference / SE	Z =1.1 / 1.25 = 0.88	
Step : 4	Decision	Accept H0 since calculated value of z (0.88) is < 1.28		
Step : 5	Conclusion	No increase in the monthly bill		



Hypothesis Testing: σ Known

Test the claim that the true mean weight of chocolate bars manufactured in a factory is 3 ounces. Suppose the sample results are n = 100, X bar = 2.84 (σ = 0.8 is assumed known from past company records). Suppose that α = .05 is chosen for this test

Testing the Population Mean (σ known) Use Z distribution

=NORM.S.DIST(-2,1)

P=0.02275

Since p is < alpha Reject H0

Data Given

No. of samples		1 (chocolate bar)
Population average	μ	3 ounce
Population SD	σ	0.8
Sample size	n	100
Sample average	$\dot{X}$	2.84
Key word	absent	
Level of significance	α	5%

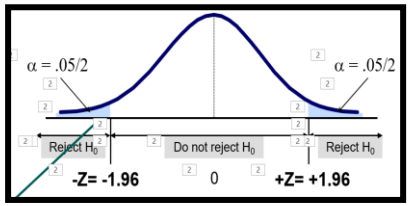
=NORM.S.INV(0.975)

Key word is absent, hence " apply 2 tail test , table value of z is 1.96

Ho :μ = 3 ounce

H1 :μ ≠ 3 ounce

Step : 1	Find the difference	$\dot{X} - \mu$	2.84-3	-0.16
Step : 2	Find the standard error	**σ / √n**	0.8 / √100	0.08
Step : 3	Test Statistics	**Z = Difference / SE**	Z = -0.16 / 0.08 =-2.	
Step : 4	Decision	Reject H0 since calculated value of z (2.) is > 1.96		
Step : 5	Conclusion	**Mean weight is not equal to 3 ounce**		



	5% LOS	1% LOS
One Tail Test	1.64	2.33
Two Tail Test	1.96	2.58

Hypothesis Testing: one sample

σ Unknown

- If the population standard deviation is unknown, you instead use the sample standard deviation S.
- Because of this change, you use the t distribution instead of the Z distribution to test the null hypothesis about the mean.
- All other steps, concepts, and conclusions are the same.

Hypothesis Testing:
σ Unknown

- Recall that the t test statistic with n-1 degrees of freedom is:

$$t_{n-1} = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Degrees of freedom are the number of independent values that a statistical analysis can estimate.

You can also think of it as the number of values that are free to vary as you estimate parameters.

Hypothesis Testing:

σ Unknown Example

The mean cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in X bar = $172.50 and S = 15.40. Test at the $\alpha = 0.05$ level.(A stem-and-leaf display and a normal probability plot indicate the data are approximately normally distributed)

Data Given

No. of samples		1 Hotel
Population average	μ	$168
Sample SD	s	15.4
Sample size	n	25
Sample average	$\dot{X}$	$ 172.5
Key word		absent
Level of significance	α	5%

Table value of t when n= 25, d.f = n-1, alpha 5% ,is 2.064

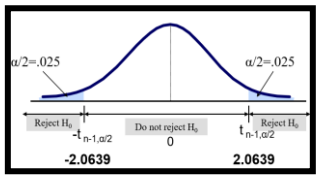
Testing the Population Mean (σ is unknown) Use t distribution

$H_0: \mu = 168 \quad H_1: \mu \neq 168$

Step : 1	Find the difference	$\dot{X} - \mu$	172.5 -168	4.5
Step : 2	Find the standard error	s / √n-1	15.4 / √25-1	3.07
Step : 3	Test Statistics	t = Difference / SE	t = 4.5 / 3.07 =1.47	
Step : 4	Decision	Accept H0 since calculated value of t (1.47) is < 2.064		
Step : 5	Conclusion	Not sufficient evidence that true mean cost is different from $168		

Using Excel

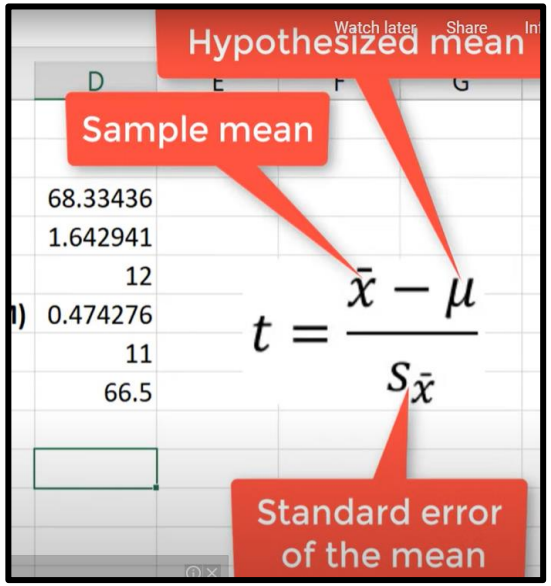
=T.INV.2T(0.05,24) 2.063899



Problem - 2

H_0: μ = 69.8 H_1: μ ≠ 69.8

	A
1	Height (inches)
2	65.78331
3	71.51521
4	69.39874
5	68.2166
6	67.78781
7	68.69784
8	69.80204
9	70.01472
10	67.90265
11	66.78236
12	66.48769
13	67.62333



	A	B	C	D	E
1	Height (inches)				
2	65.78331				
3	71.51521		Mean (x̄)	68.33436	=AVERAGE(A2:A13)
4	69.39874		Standard deviation (s)	1.642941	=STDEV(A2:A13)
5	68.2166		Count (n)	12	=COUNT(A2:A13)
6	67.78781		Standard error of mean (SEM)	0.474276	=D4/SQRT(D5)
7	68.69784		Degrees of freedom (df)	11	=D5-1
8	69.80204		Hypothesized mean (μ)	66.5	
9	70.01472				
10	67.90265		t-statistic		=(D3-D8)/D6
11	66.78236		p-value	0.002618	

Conclusion : National Avg Height ≠ 69.80

Decision : since p value is < α, reject H0

Hypothesis Testing: σ Unknown

- Recall that you assume that the sample statistic comes from a random sample from a normal distribution.
- If the sample size is small (< 30), you should use a box-and-whisker plot or a normal probability plot to assess whether the assumption of normality is valid.
- If the sample size is large, the central limit theorem applies and the sampling distribution of the mean will be normal.

Hypothesis Testing

Proportions

- Involves categorical variables
- Two possible outcomes
  - “Success” (possesses a certain characteristic)
  - “Failure” (does not possesses that characteristic)
- Fraction or proportion of the population in the “success” category is denoted by π

Hypothesis Testing
Proportions

- Sample proportion in the success category is denoted by p

$$p = \frac{X}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$

- When both $n\pi$ and $n(1-\pi)$ are at least 5, p can be approximated by a normal distribution with mean and standard deviation

$$\mu_p = \pi$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

Hypothesis Testing
Proportions

- The sampling distribution of p is approximately normal, so the test statistic is a Z value:

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$

Hypothesis Testing

Proportions Example

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 30 responses. Test at the $\alpha = .05$ significance level.

Check whether $n\pi$ and $n(1-\pi)$ are atleast 5

$n\pi = (500)(.08) = 40$

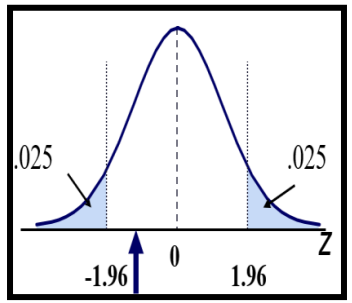
$n(1-\pi) = (500)(.92) = 460$

Can be approximated by normal distribution

Data Given

π =Proportion of responses	0.08
No of Sample	500
Sample responses	30
p=Sample proportion	30/500 =0.06
Key word	absent

Table value of z for 2 tailed test and 5% Level of significance is 1-.96



Using Excel

=NORM.S.INV(0.975) **1.959964**

$H_0: \pi = 0.08$ $H_1: \pi \neq 0.08$

Step : 1

Find the difference | **$|p - \pi|$** | **$|0.06 - 0.08|$** | **0.02**

Step : 2

Find the standard error $\sqrt{\frac{\pi(1-\pi)}{n}}$ $\sqrt{\frac{.08(1-.08)}{500}}$ **0.0121**

Step : 3

Test Statistics | **Z = Difference / SE** | **Z= 0.02 / 0.0121 =1.648**

Step : 4

Decision | **Accept H0 since calculated value of Z (1.642) is < 1.96**

Step : 5

Conclusion | There isn't sufficient evidence to reject the company's claim of 8% response rate.

Hypothesis Testing – Problem

A manufacturer claims that at least 95% of the equipment that is supplied to a factory conforms to the specifications. An examination of 200 pieces of equipment revealed that 18 were faulty. Test this claim at a significant level of 1% and 5% (Proportion of the same kind alone to be compared)

Check whether n π and n(1-π) are atleast 5

n π = (200)(.05) = 10

n(1-π) = (200)(.95) = 190

Can be approximated by normal distribution

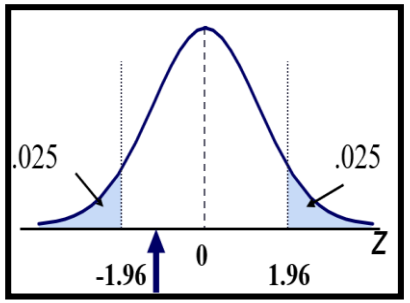
H_0: π = 0.05 H_1: π ≠ 0.05

Data Given

π =Proportion of faulty eqpt	1-0.95 =0.05
No of Sample	200
Faulty items	18
p= proportion of defective	18/200 =0.09
Key word	absent
LOS	5% & 1%

Table value of z for 2 tailed test - for 5% Level of significance is 1.96 & for 1 % LOS is 2.58

For 5 %	=NORM.S.INV(0.975)	= 1.959964
For 1 %	=NORM.S.INV(0.995)	=2.575829



Step : 1

Find the difference | p- π | | 0.09 – 0.05 | **0.04**

Step : 2

Find the standard error $\sqrt{\pi(}\ \frac{0.05 * .95}{200}$ $\sqrt{\frac{.08(1-.08)}{500}}$ **0.0154**

Step : 3

Test Statistics **Z = Difference / SE** **Z= 0.04 / 0.0154 =2.597**

Step : 4

Decision **Reject H0 since calculated value of Z (2.597) is > 1.96 (5%) & 2.58 (1%)**

Step : 5

Conclusion Claim is not valid

Hypothesis Testing – Problem

A sales clerk in the departmental store claims that 60% of the shoppers entering the store leave without making a purchase. A random sample of 50 shoppers showed 35 of them left without buying anything .Are these sample results consistent with the claim of the sales clerk?

Check whether n π and n(1-π) are atleast 5

n π = (50)(0.60) = 30

n(1-π) = (50)(1-.60)= 20

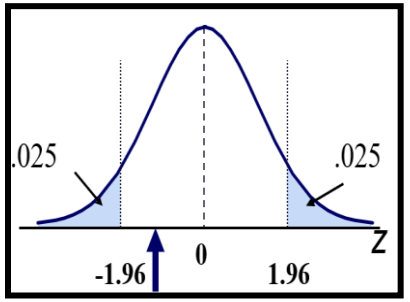
Can be approximated by normal distribution

Data Given

π =Proportion of not buying	0.60
No of Sample	50
Not buying	35
p= proportion of not buying	35/50 = 0.70
Key word	absent
LOS	5%

Table value of z for 2 tailed test and 5% Level of significance is 1.96

For 5 %	=NORM.S.INV(0.975)	= 1.959964



H_0: π = 0.60 H_1: π ≠ 0.60

Step : 1

Find the difference | p- π | | 0.70 – 0.60 | **0.10**

Step : 2

Find the standard error $\sqrt{\frac{0.60 * .0.4}{50}}$ $\sqrt{\frac{.08(1-.08)}{500}}$ **0.069**

Step : 3

Test Statistics **Z = Difference / SE** **Z= 0.10 / 0.069 =1.45**

Step : 4

Decision **Accept H0 since calculated value of Z (1.45) is < 1.96**

Step : 5

Conclusion Sles clerk Claim is t valid

Potential Pitfalls and Ethical Considerations

- Use randomly collected data to reduce selection biases
- Do not use human subjects without informed consent
- Choose the level of significance, α, before data collection
- Do not employ “data snooping” to choose between one-tail and two-tail test, or to determine the level of significance
- Do not practice “data cleansing” to hide observations that do not support a stated hypothesis
- Report all pertinent findings

Chapter Summary

We have

- Addressed hypothesis testing methodology
- Performed Z Test for the mean (σ known)
- Discussed critical value and p–value approaches to hypothesis testing
- Performed one-tail and two-tail tests
- Performed t test for the mean (σ unknown)
- Performed Z test for the proportion
- Discussed pitfalls and ethical issues