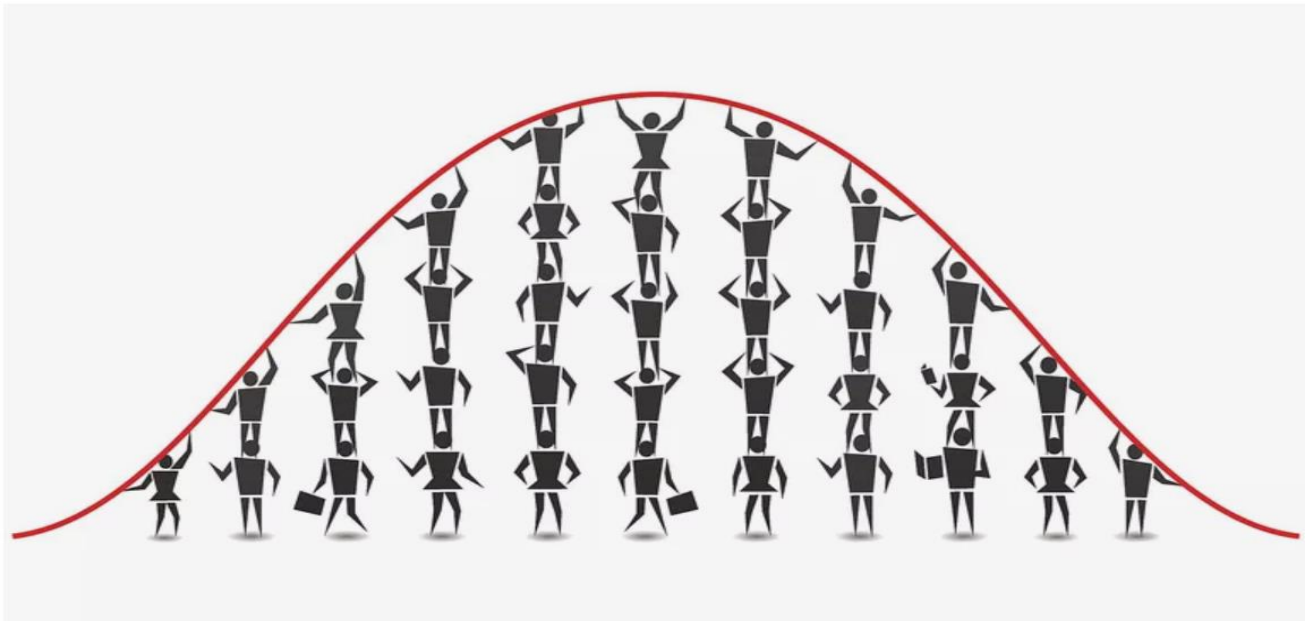


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Continuous Probability Distributions



NORMAL DISTRIBUTION

A normal distribution of data is one in which the majority of data points are relatively similar, meaning they occur within a small range of values with fewer outliers on the high and low ends of the data range.

When data are normally distributed, plotting them on a graph results a bell-shaped and symmetrical image often called the bell curve.
In such a distribution of data, mean, median, and mode are all the same value and coincide with the peak of the curve.

Learning Objectives

You will learn:

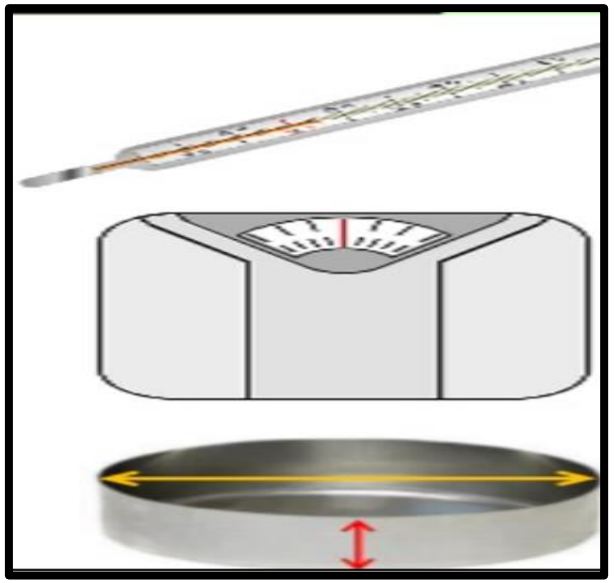
- To compute probabilities from the normal distribution.
- To use the normal probability plot to determine whether a set of data is approximately normally distributed.

Continuous Probability Distributions

- A continuous random variable is a variable that can assume any value on a continuum (can assume an uncountable number of values)

Examples :

1. Body Temperature
2. Heights & weights of Adults
3. The thickness and dimensions of a product
4. IQ and standardized test scores
5. Quality control test results
6. Errors in measurement

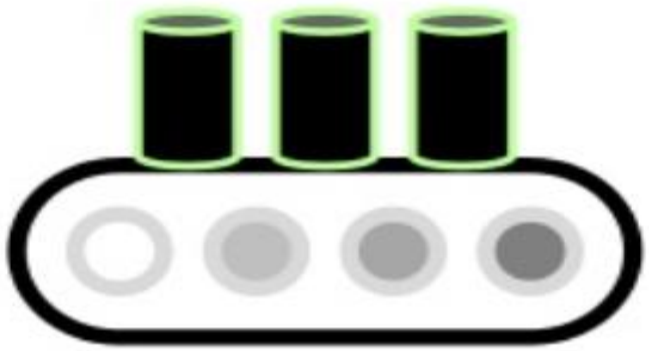


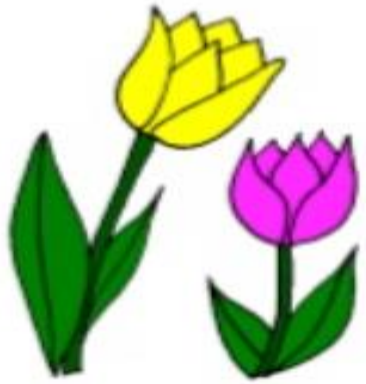
- These can potentially take on any value, depending only on the ability to measure precisely and accurately.

To find the Probability of a continuous random variable (Measurable data)

- Normal Distribution

- Can be found practically everywhere:
  - In nature.
  - In engineering and industrial processes.
  - In social and human sciences.
- Many everyday data sets follow approximately the normal distribution.

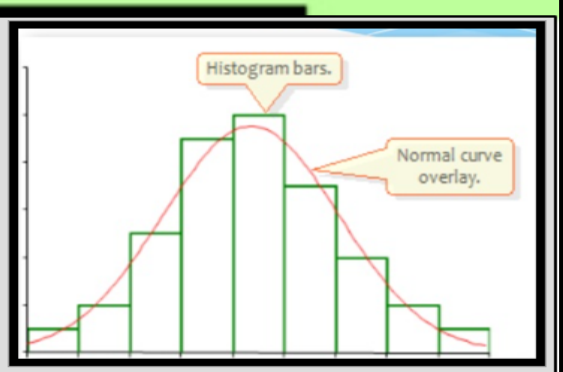






- Normal Distribution

Why ?



- Used to illustrate the shape and variability of the data
- Used to estimate future process performance
- Normality is an important assumption when conducting Statistical analysis
- Certain SPC charts and many statistical inference tests require the data to be normally distributed

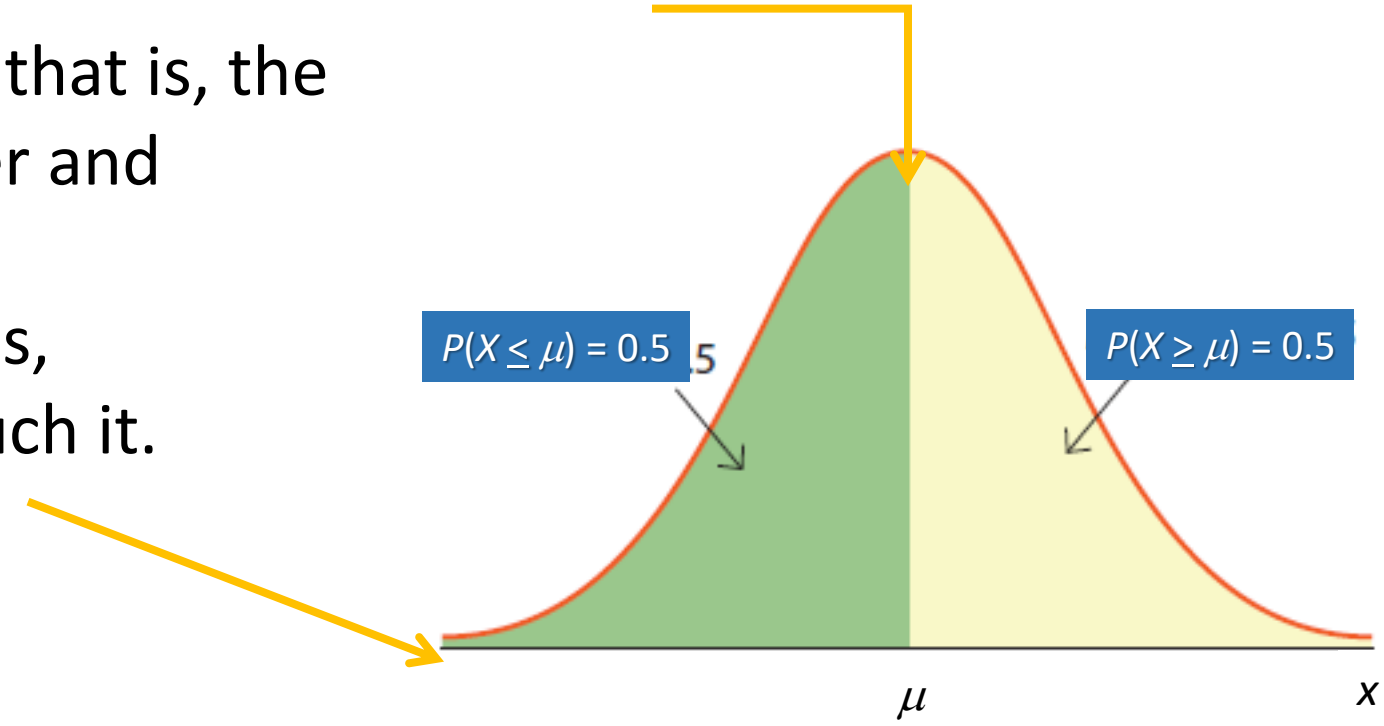
The Normal Distribution

Explain the characteristics of the normal distribution.

- The **Normal Distribution**
  - ✓ Closely approximates the probability distribution of a wide range of random variables, such as the:
    - Heights and weights of newborn babies
    - Scores on SAT
    - Cumulative debt of college graduates
  - ✓ Serves as the cornerstone of statistical inference.

Normal Distribution

- Characteristics of the **Normal Distribution**
  - ✓ **Bell-shaped** and **Symmetric** about its mean
    - Mean = Median = Mode
  - ✓ **Asymptotic** - that is, the tails get closer and closer to the horizontal axis, but never touch it.



The Normal Distribution Density Function

- The formula for the normal probability density function is

$$f(X) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{(X-\mu)}{\sigma}\right)^2}$$

Where e = the mathematical constant approximated by 2.71828

π = the mathematical constant approximated by 3.14159

μ = the population mean

σ = the population standard deviation

X = any value of the continuous variable

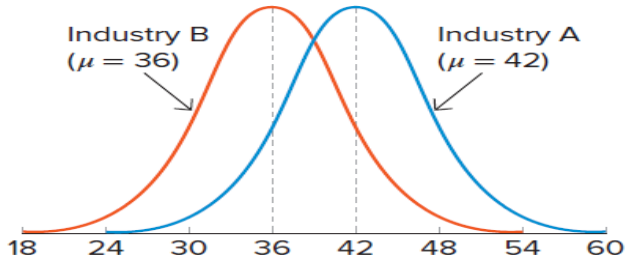
Normal Distribution

Example: Suppose the ages of employees in Industries A, B, and C are normally distributed.

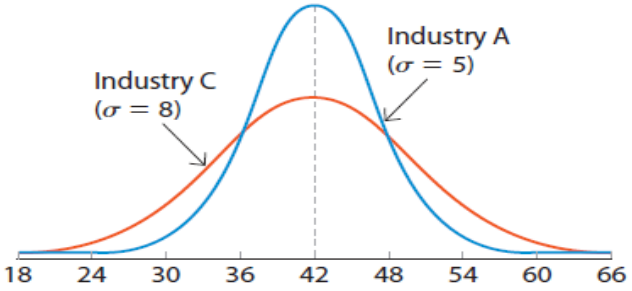
✓ The relevant parameters are:

Industry A	Industry B	Industry C
$\mu = 42$ years	$\mu = 36$ years	$\mu = 42$ years
$\sigma = 5$ years	$\sigma = 5$ years	$\sigma = 8$ years

✓ Compare industries using the Normal curves:



σ is the same, μ is different.



μ is the same, σ is different.

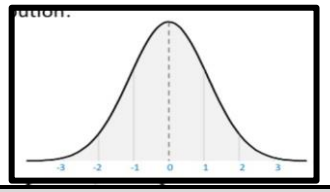
- Normal Distribution

- Properties of Normal Distribution

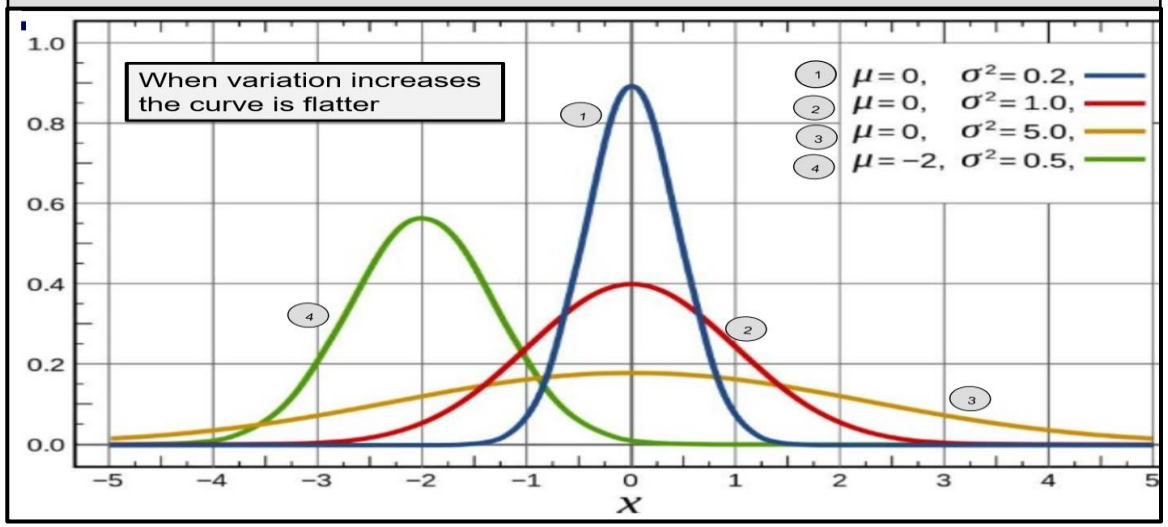
1. Bell Shaped
2 Symmetrical
3. Mean, Median and Mode are equal
4 Location is characterized by the mean, μ
5 Spread is characterized by the standard deviation, σ
6 The random variable has an infinite theoretical range: -∞ to +∞
7 It is unimodal
8 μ ± 1 σ will cover 68.26 % area , μ ± 2 σ will cover 95.43 % area & μ ± 3 σ will cover 99.73 % area
9 For std Normal curve , Mean 0 & s.d is 1

Standard Normal Distribution:

- A common practice to convert any normal distribution to the standardized form and then use the standard normal table to find probabilities.
- The **Standard Normal Distribution** (Z distribution) is a way of standardizing the normal distribution.
- It always has a mean of **0** and a standard deviation of **1**.

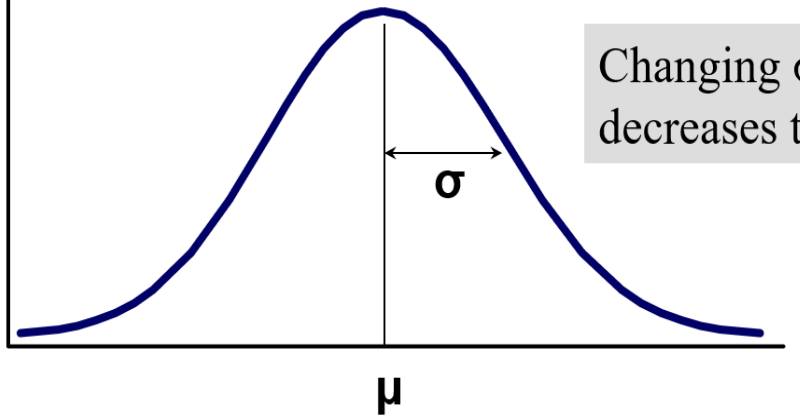


By varying the parameters μ and σ, we obtain different normal distributions



f(X)

Changing **μ** shifts the distribution left or right.



Changing σ increases or decreases the spread.

Normal Distribution

Calculate and interpret probabilities for a random variable that follows the normal distribution.

- The **Normal Transformation**
  - ✓ Any normally distributed random variable X with mean μ and standard deviation σ can be transformed into the standard normal random variable Z as:

$$Z = \frac{X - \mu}{\sigma}$$ with corresponding values $$z = \frac{x - \mu}{\sigma}$$

  - ✓ By construction: $E(Z) = 0$ and $SD(Z) = 1$.

Normal Distribution

- **Standard Normal Table (*z* Table)**.
  - ✓ Gives the cumulative probabilities $P(Z \leq z)$ for positive and negative values of z.
  - ✓ Since the random variable Z is symmetric about its mean of 0,

$$P(Z \leq 0) = P(Z \geq 0) = 0.5.$$

  - ✓ To obtain the $P(Z \leq z)$, read down the z column first, then across the top.

Normal Distribution

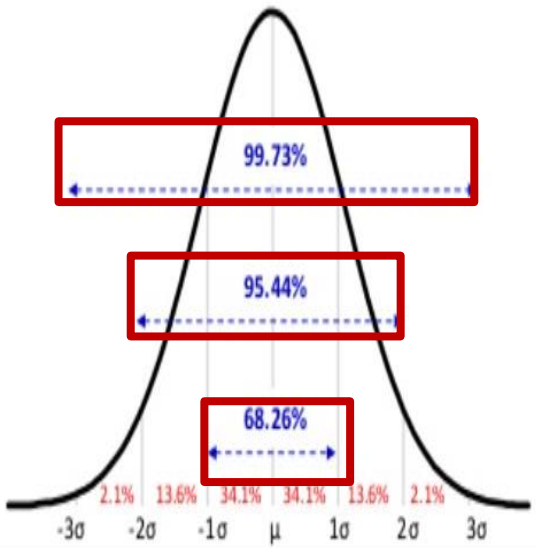
- A z value specifies by how many standard deviations the corresponding x value falls above ($z > 0$) or below ($z < 0$) the mean.

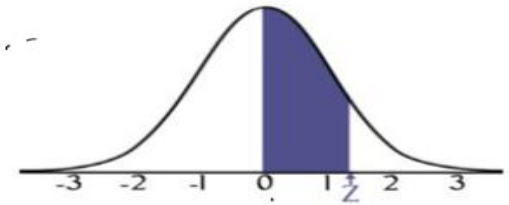
  - ✓ A positive z indicates by how many standard deviations the corresponding x lies above μ.
  - ✓ A zero z indicates that the corresponding x equals μ.
  - ✓ A negative z indicates by how many standard deviations the corresponding x lies below μ.

- Normal Distribution

Empirical Rule:

- For any normally distributed data:
  - **68%** of the data fall within **1** standard deviation of the mean.
  - **95%** of the data fall within **2** standard deviations of the mean.
  - **99.7%** of the data fall within **3** standard deviations of the mean.





STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and *z* standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	[illegible]	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	[illegible]	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

To Find the Area under Normal curve for various Z value Using Excel

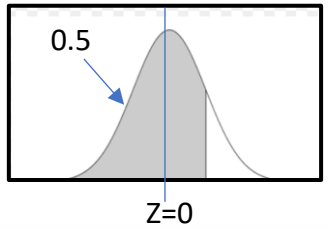
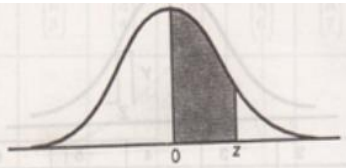


TABLE -2

TABLE -1



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.01	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.055	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.094	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.133	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1519	.1628	.1664	.170	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.205	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.239	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.270	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.299	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.326	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.350	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.372	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.392	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.409	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.424	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.438	.4394	.4406	.4418	.4429	.4441
1.6					.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

When z=1

=NORM.S.DIST(1,TRUE)

0.841345

When z=1.64

=NORM.S.DIST(1.64,TRUE)

0.949497

When z=1.09

=NORM.S.DIST(1.09,TRUE)

0.862143

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

The Normal Distribution

- Characteristics of the **Normal Distribution**
  - ✓ The normal distribution is **completely described by two parameters**: μ and σ^2.
    - μ is the population mean which describes the central location of the distribution.
    - σ^2 is the population variance which describes the dispersion of the distribution.

The Standardized Normal Distribution: Density Function

- The formula for the standardized normal probability density function is

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}}$$

Where e = the mathematical constant approximated by 2.71828

π = the mathematical constant approximated by 3.14159

Z = any value of the standardized normal distribution

Normal Distribution

- **Example:** Finding Probabilities for a Standard Normal Random Variable *Z*.

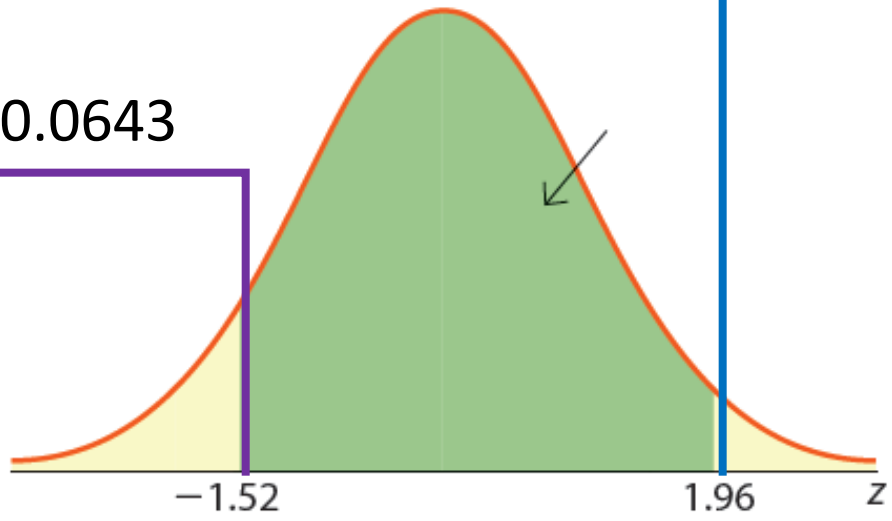
✓ Find $P(-1.52 \leq Z \leq 1.96)$ =

$P(Z \leq 1.96) - P(Z \leq -1.52) =$

$P(Z \leq 1.96) = 0.9750$

$P(Z \leq -1.52) = 0.0643$

$0.9750 - 0.0643 = 0.9107$



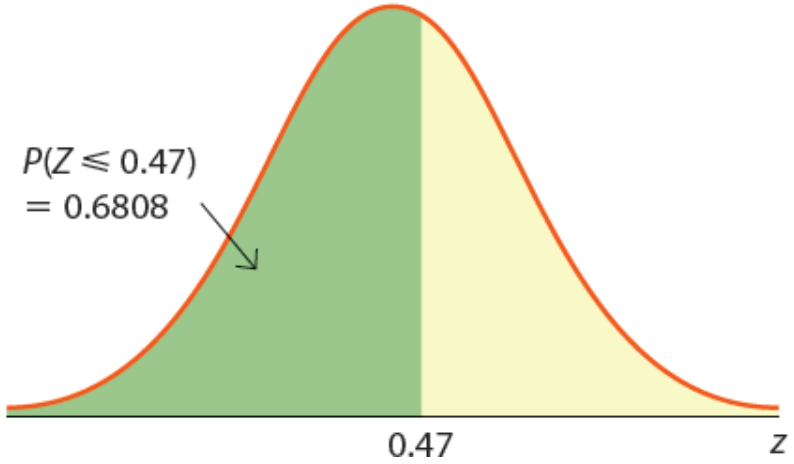
USING EXCEL

	A	B	C
1	$P(X \leq 1.96) =$	0.9750	=NORM.S.DIST(1.96,1)
2	$P(X \leq -1.52) =$	0.0643	=NORM.S.DIST(-1.52,1)
3	$P(-1.52 \leq X \leq 1.96) =$	0.9107	=B1-B2

Normal Distribution

- Example: Use Excel to find a *z* value for a given probability.
  - ✓ **For a standard normal variable *Z*, find the *z* values that satisfy $P(Z \leq z) = 0.6808$.**

	A	B
1	0.47	=NORM.S.INV(0.6808)



Normal Distribution

- Use the **Inverse Transformation** to compute probabilities for given *x* values.

  ✓ The standard normal variable *Z* can be transformed to the normally distributed random variable *X* with mean μ and standard deviation σ as:

$$X = \mu + Z\sigma$$ with corresponding values $$x = \mu + z\sigma$$

Normal Distribution - EXAMPLE

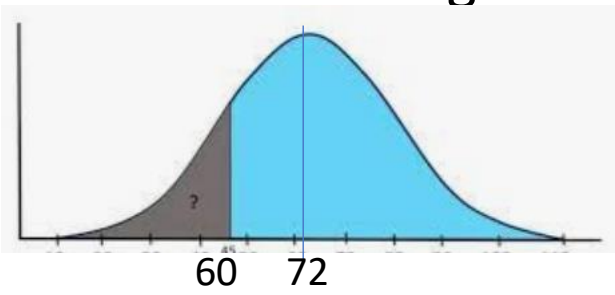
Scores on a management aptitude exam are normally distributed with a mean of 72 (μ) and a standard deviation of 8 (σ).

What is the probability that a randomly selected manager will score above 60?

- METHOD 2 :First transform the random variable *X* to *Z* using the transformation formula:

$$z = \frac{x - \mu}{\sigma} = \frac{60 - 72}{8} = -1.5$$

- Using the standard normal table, find



$P(Z > -1.5)$ IS SAME AS P(Z <1.5)

THE TABLE VALUE FOR Z=1.5 IS 0.9332

P(X > 60)

USING EXCEL : =1-NORM.DIST(60,72,8,1)

P(X > 60) = 0.9332

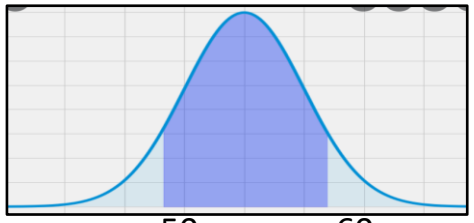
	A	B	C
1	$P(X \leq 60)$ =	0.0668	=NORM.DIST(60,72,8,1)
2	$P(X > 60)$ =	0.9332	=1-B1

APPLICATION OF NORMAL DISTRIBUTION

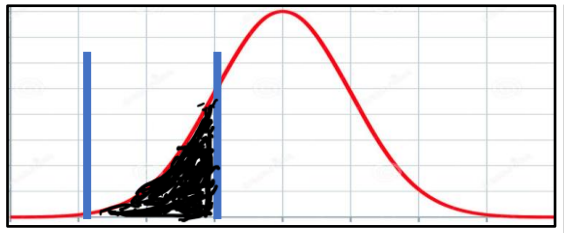
Job Title	Location	Mean Annual Salary	Standard Deviation Annual Salary
Accounting	Seattle, WA	$53,000	$4,700
Accounting	USA	$66,000	$6,200
Accounting	Oakland, CA	$58,000	$5,600

Calculation	Seattle, WA	USA	Oakland, CA
Mean	$53,000	$66,000	$58,000
SD	$4,700	$6,200	$5,600
x lower	$50,000	$50,000	$50,000
x upper	$60,000	$60,000	$60,000
Operator	between		
P(50000 <= x <= 60000)	0.670164	0.161656	0.562944

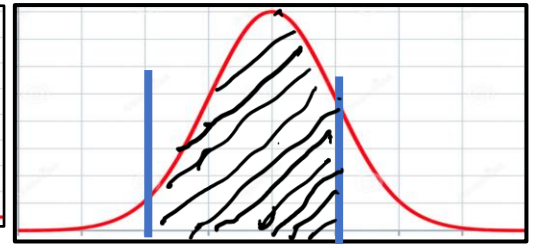
What is the probability of getting a job between 50,000 to 60,000 dollars in the following cities ,if mean & s.d is known for each city
(1) Seattle (2) USA (3) Oakland



σ =4.7



σ =6.2



σ =5.6

= NORM.DIST(60,53,4.7,1)-NORM.DIST(50,53,4.7,1)

Example

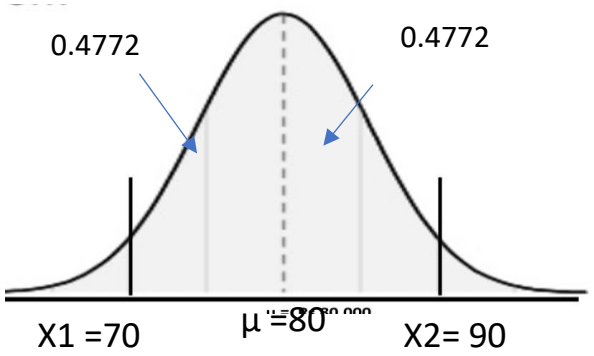
- Average salary offered to student in Great lake is Rs 80,000 p.m standard deviation is Rs 5000 What is the probability of getting a salary between Rs 70,000 & Rs 90,000

Data Given: μ = Rs 80,000 σ =5,000

Find Z score when x= Rs 70,000

$$Z = \frac{x-\mu}{\sigma} = \frac{70-80}{5} = -2$$

Table value when z =2 is 0.4772



To Find Z score when x= Rs 90,000

$$Z = \frac{x-\mu}{\sigma} = \frac{90-80}{5} = 2$$

Table value when z = 2 is 0.4772

Using Excel

=NORM.DIST(90,80,5,TRUE)-NORM.DIST(70,80,5,TRUE)

$P(70 \le x \le 90) = 0.9545$

P(70 ≤ x ≤ 90) = Probability of getting a salary between Rs 70,000 & Rs 90,000 =0.4772+ 0.4772 =0.9544
(i.e) 95.44 % chance of getting a salary between Rs 70,000 & Rs 90,000

Problem

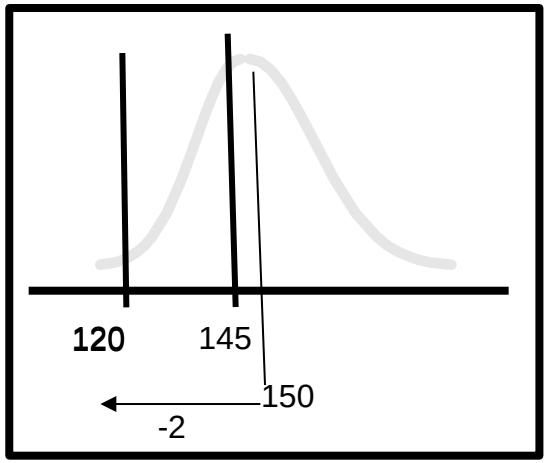
- The average daily sales of 500 branch ofices was Rs 150 thousands and the standard deviation Rs 15 thousands .Assuming the distribution to be normal , indicate how many branches have sales between
- (1) Rs 120 thousand and Rs 145 thousands (2)Rs 140 & Rs 165 thousands

Data Given: $\mu = 150$ $\sigma = 15$

Z score when x= 120

$$Z = \frac{x-\mu}{\sigma} = \frac{120-150}{15} = -2$$

Table value when z =2 is 0.4772



Z score when x= 145

$$Z = \frac{x-\mu}{\sigma} = \frac{145-150}{15} = -0.33$$

Table value when z =0.33 is 0.1293

Using Excel

=NORM.DIST(145,150,15,TRUE)-NORM.DIST(120,150,15,TRUE)

=0.346691

No of Branches= 600 * 0.346691 = 173

Expected no of branches having sales between Rs 120 & 145 thousands
= 500 * (0.4772-0.1293) = 500* 0.3479 =173.93 say 174 branches

PROBLEM

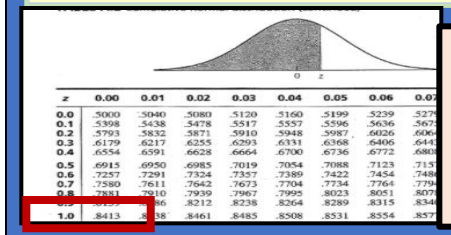
You score 80 % in QT 1 assessment test and your close friend also scored 80 % on their test in another section. If the average of both the tests is 75 and the standard deviation is 5 and 10 respy . Who do you think did better on their test? Why do you think so ?

Z score for your test

Data Given:

X= 80 $\mu = 75$ $\sigma = 5$

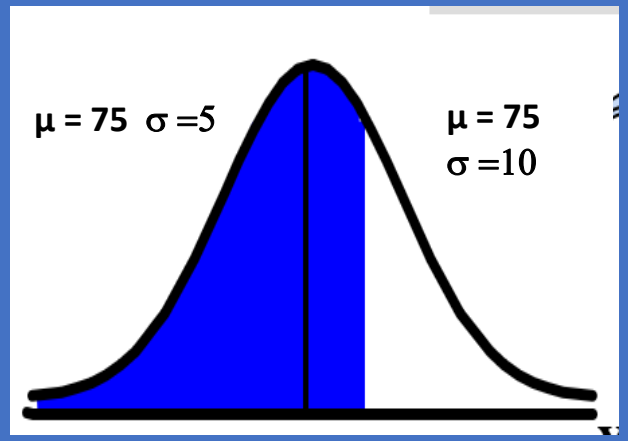
$$Z = \frac{x-\mu}{\sigma} = \frac{80-75}{5} = 1$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577

The table value is 0.8413

Your mark is at the 84 th percentile



What if your Mark is 55%
1.Calculate the Z score ?
2.What % of students have a score below your score & above your score

Z score for your Friend's test

Data Given:

X= 80 $\mu = 75$ $\sigma = 10$

$$Z = \frac{x-\mu}{\sigma} = \frac{80-75}{10} = 0.5$$

The table value is 0.6915

Your friends mark is at the 69th percentile

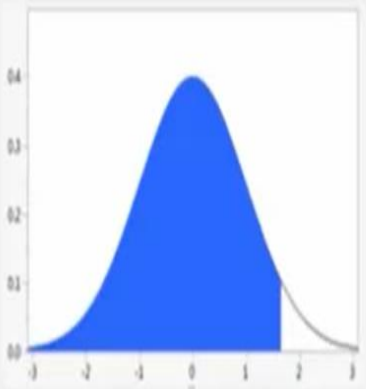
WHAT IS THE Z VALUE?

What test score will place you in the top 95th percentile?

Average (μ) = 500

Standard deviation (σ) = 100

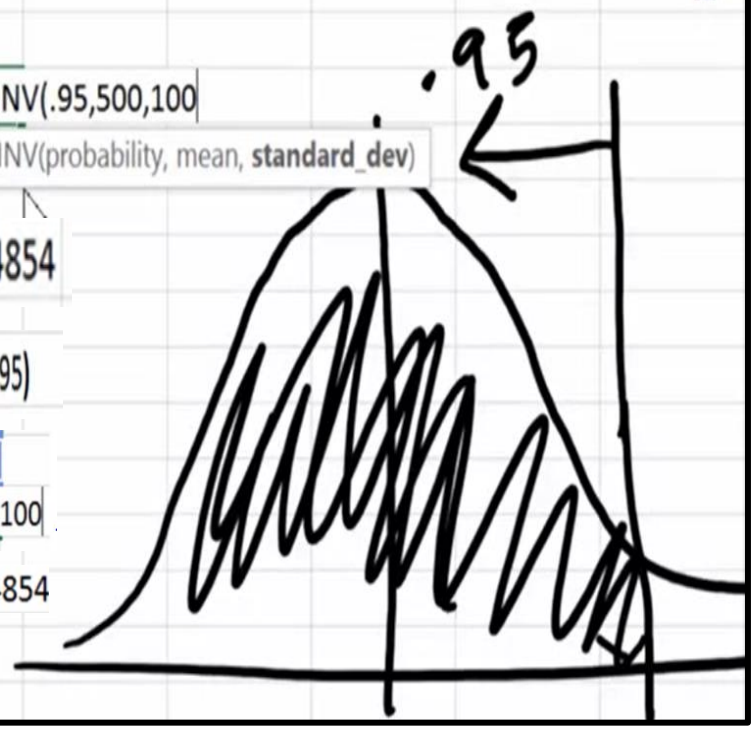
$P(x \leq X) = 0.95$



2-5.7. Finding Z for a Given Probability (INV) in Excel

Share

	G	H	I	J	K	L	M
	score at 95%	=NORM.INV(.95,500,100					
		NORM.INV(probability, mean, **standard_dev**)					
	score at 95%	664.4854					
	z-score	=NORM.S.INV(.95)					
	z-score	1.644854					
	score	=500+(H6*100					
	score	664.4854					

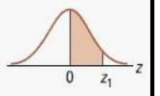


Problem

- A wholesale distributor of fertilizer products finds that the annual demand for one type of fertilizer is normally distributed with a mean of 120 tonnes and standard deviation of 16 tonnes . If he orders only once a year,what quantity should be ordered to ensure that there is only a 5 percent chance of running short ?

TABLE A.5 Areas of the standard normal distribution $\mu = 0$, $\sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and z_1 (the shaded area).



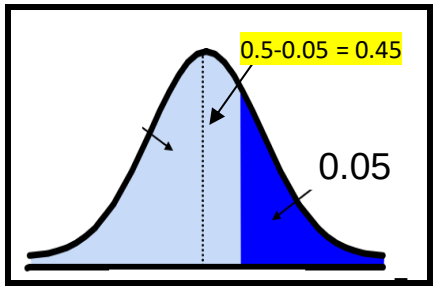
z_1	0.00	0.01	0.02	0.03	0.04	0.05
0.0	.0000	.0040	.0080	.0120	.01[illegible]	.0199
0.1	.0398	.0438	.0478	.0517	.05[illegible]	.0596
0.2	.0793	.0832	.0871	.0910	.09[illegible]	.0987
0.3	.1179	.1217	.1255	.1293	.13[illegible]	.1368
0.4	.1554	.1591	.1628	.1664	.17[illegible]	.1736
0.5	.1915	.1950	.1985	.2019	.20[illegible]	.2088
0.6	.2257	.2291	.2324	.2357	.2389	.2422
0.7	.2580	.2611	.2642	.2673	.2704	.2734
0.8	.2881	.2910	.2939	.2967	.2995	.3023
0.9	.3159	.3186	.3212	.3238	.3264	.3289
1.0	.3413	.3438	.3461	.3485	.3508	.3531
1.1	.3643	.3665	.3686	.3708	.3729	.3749
1.2	.3849	.3869	.3888	.3907	.3925	.3944
1.3	.4032	.4049	.4066	.4082	.4099	.4115
1.4	.4192	.4207	.4222	.4236	.4251	.4265
1.5	.4332	.4345	.4357	.4370	[illegible]	.4394
1.6	.4452	.4463	.4474	.4484	.4495	.4505
1.7	.4554	.4564	.4573	.4582	.4591	.4599

When area is 0.45 , the table value of z is 1.64

Data Given: $\mu = 120$ $\sigma = 16$

$$Z = \frac{x-\mu}{\sigma}$$

Find x when z = 1.64



$$Z = 1.64 = \frac{x-120}{16}$$

x= 120+16*1.64 = 146.24

X =NORM.INV(0.95,120,16) = 146.3177

Wholesale distributor should order 147 units

Problem

- There are 600 business students in the graduate department of a university , and the probability for any student to need a copy of a particular text book from the library on any day is 0.05 . How many copies of the book should be kept in the university library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed (Use Normal approximation to the binomial probability law)

Data Given: $\mu = n*p = 600*0.05 = 30$

$\sigma = \sqrt{npq} = \sqrt{600 * 0.05 * 0.95} = 5.3$

When area is 0.40 , the table value of z is 1.28

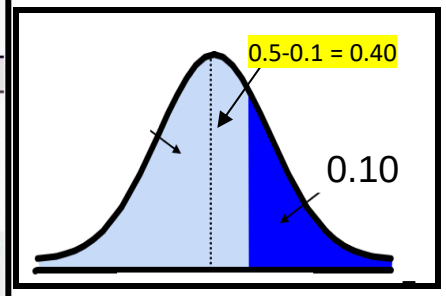
$$Z = \frac{x-\mu}{\sigma}$$

TABLE A.5 Areas of the standard normal distribution $\mu = 0, \sigma = 1$

The entries in this table are the probabilities that a standard normal random variable is between 0 and z_1 (the shaded area).



z_1	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	[illegible]19	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	[illegible]14	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	[illegible]03	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	[illegible]80	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	[illegible]44	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	[illegible]10	.3830
1.2	[illegible]45	[illegible]69	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177



Find x when z = 1.28

$$Z = 1.28 = \frac{x-30}{5.3}$$

x= 30+5.3*1.28 = 36.784

Using Excel :

X =NORM.INV(0.9,30,5.3) = 36.79222

37 copies of the text book are required any day

Normal Distribution - Example

Akiko is concerned about buying the right amount of salmon for Little Ginza. The daily consumption is normally distributed with a mean of 12 pounds and a standard deviation of 3.2 pounds.

- What proportion of days was demand above 20 pounds?
- What proportion of days was demand below 15 pounds?

$$P(x > 20) = P\left(Z > \frac{20 - 12}{3.2}\right) = P(Z > 2.5) = 1 - P(Z \leq 2.5) = 1 - 0.9938 = 0.0062$$

$$P(x < 15) = P\left(Z < \frac{15 - 12}{3.2}\right) = P(Z < 0.94) = 0.8264$$

Normal Distribution Example

- Akiko is concerned about buying the right amount of salmon for Little Ginza. The daily consumption is normally distributed with a mean of 12 pounds and a standard deviation of 3.2 pounds.
  - How much salmon should be bought so that it meets customer demand on 90% of the days?

In order to answer this, solve for the *z*-score of a 90% probability and then plug that *z* into the equation: $x = \mu + z\sigma$

$$P(Z \leq z) = 0.90 \Rightarrow z = 1.28$$

$$x = 12 + 0.28 \cdot 3.2 = 16.10$$

Normal Distribution - Example

- Akiko is concerned about buying the right amount of salmon for Little Ginza. The daily consumption is normally distributed with a mean of 12 pounds and a standard deviation of 3.2 pounds.
  - How much salmon should be bought so that it meets customer demand on 90% of the days?

- Using Excel:

	A	B	C
1	**x** =	16.10	=NORM.INV(0.9,12,3.2)